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Measuring the extent of synergies among innovation actors and their contributions: the Helix as a cooperative game

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ABSTRACT

We generalize the 3-player game introduced by Mêgnigbêto (2018) to analyze the synergies existing between universities, the industry and the government in the Triple Helix, a model of research and innovation introduced by Leydesdorff and Etzkowitz (1995). We consider situations involving any number of actors and we allow for a differentiation of their contributions. The resulting game has nonnegative Harsanyi dividends, implying its convexity. The relative size of the core measures the extent of the synergies and the Shapley value measures the contribution of each actor to these synergies. Incidentally, the resulting game lends itself to a wide range of applications.

KEYWORDS

Innovation; transferable utility games; core; Shapley value

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1. Introduction

The analysis of the synergies existing between universities, industry and government within research and innovation models began with the Triple Helix introduced by Leydesdorff and Etzkowitz (1995): universities conduct research, industry develops products and markets them, and the government imposes normative control, but each actor can also partly play the role of another. The actors interact in pairs within three different double helices and, at a higher level, the helices combine to create synergy, lay and maintain production and knowledge sharing infrastructures, with the aim of creating and disseminating innovation.

Even if the research has focused on publications as a tangible indicator of collaboration, other elements have been considered such as patents, grants and, more generally, any element likely to fuel the relations between the different actors. The Triple Helix was later analyzed as a 3-player cooperative game by Mêgnigbêto (2018), proposing different solutions concepts aimed at measuring the extent of the collaborations and the contributions of each actor within the Triple Helix.[1]

Many researchers have called for an extension of the model beyond the three initial actors, by including for instance the civil society, financial institutions, innovation users and the natural environment: they spoke of quadruple helix and quintuple helix (Carayannis & Campbell, 2010) or n-tuple helix (Leydesdorff, 2012). In line with these enlargments, the objective of the present paper is to extend the innovation game to any number of actors and to allow for a differentiation of the elements taken into account. The existing literature indeed makes no difference: the synergies are simply evaluated by counting. The introduction of differentiation may be justified. For instance, publications are not equivalent, some having been peer-reviewed and other not; patents are not equivalent, some facing competition, due to the existence of interchangeable alternatives, and others being unique contributions to a technology.[2]

The paper is structured as follows. Section 2 recalls basic definitions and properties of transferable utility game while Section 3 is devoted to solution concepts. Section 4 introduces the innovation game and its properties. An example is analyzed in Section 5 and concluding remarks are offered in the last section.

2. Transferable utility games: definitions and properties

A *transferable utility game* is a pair (N,v) where *N* is a set of *players* and *v* is a "characteristic" function that associates a real number to each coalition: $v(S)$ is the worth of coalition *S*. It measures the amount that each coalition is sure to obtain if it decided to form. By convention, we set $v(\varnothing)=0$. We denote by $G(N)$ the set of characteristic functions defined on the set of players *N*.

Given two games (N,v_1) and (N,v_2) on a common set *N* of players, the game sum (N,v_1+v_2) is defined by $(v_1+v_2)(S)=v_1(S)+v_2(S)$ for all $S\subset N$. A game can be restricted to a subset of players: the restriction of a game (N,v) to a subset $T\subset N$ is defined by the subgame (N,v_T) where $v_T(S)=v(S)$.

Cooperative game theory is concerned by the allocation of the $v(N)$, the worth of the "grand coalition", among the players, knowing what each coalition could or should obtain by itself as measured by its worth.[3] An *allocation* is a vector $x=(x_1,\dots,x_n)$, where x_i is the amount allocated to player *i*, such that

[1] Game theory has long been used to study innovation, typically in a non-cooperative perspective. See Baniak & Dubina (2012) for a review of this literature.

[2] See Layne-Farrar et al. (2007) and Dehez & Poukens (2014).

[3] The notion of characteristic function was introduced by von Neumann and Morgenstern (1944). See Aumann (2010) who makes a distinction between what a coalition could claim vs what it could get, the latest referring implicitely to some underlying strategic form game.

$x_1 + x_2 + \ldots + x_n = v(N)$. Imposing equality is nothing but a requirement of *collective rationality* or *efficiency*. Given a game (N, v), the set of allocations is defined by:

$$X(N,v) = \left\{ x \in \mathbb{R}^n \,\middle|\, \sum_{i \in N} x_i = v(N) \right\}.$$

Notation: Finite sets are denoted by upper-case letters. Lower-case letters are used to denote their cardinals: $t = |T|$, $s = |S|$, … Set inclusion is denoted by $\subset$ and strict inclusion is denoted by $\subsetneq$. We define $\mathcal{C}(S) = \{T \subset S \mid T \neq \varnothing\}$ and $\mathcal{C}_i(S) = \{T \subset S \mid i \in T\}$. Braces are sometimes omitted in the identification of coalitions: for instance, $v(i, j)$ replaces $v(\{i, j\})$. For a given vector *x*, we denote by $x(S)$ the sum of the components whose coordinates are elements of the set *S*. By convention, we set $x(\varnothing) = 0$.

It is generally assumed that disjoint coalitions never lose by merging:

$$S \cap T = \varnothing \;\Rightarrow\; v(S \cup T) \geq v(S) + v(T).$$

This is the assumption of *superadditivity*. We denote by $SG(N)$ the set of superadditive characteristic functions defined on the set of players *N*. Assuming that the individual worth $v(i)$ are all nonnegative, superadditivity implies nonnegativity for all $v(S)$ and *monotonicity*: $S \subset T \;\Rightarrow\; v(S) \leq v(T)$. Convexity is more restrictive than superadditivity. A game is *convex* if the following inequalities hold

$$v(S \cup T) \geq v(S) + v(T) + v(S \cap T) \quad \text{for all } \; S, T \subset N. \tag{1}$$

Clearly, convexity implies superadditivity.[4] A convex game is *strictly convex* if (1) holds with a strict inequality whenever $S \not\subset T$ nor $T \not\subset S$.

Players' contributions to coalitions play a central role in allocation problems. The *marginal contribution* of player *i* to coalition *S* is defined by $MC_i^v(S) = v(S) - v(S \setminus i)$. It is nonnegative if the game is monotonic and, of course, it is null if $i \notin S$. Shapley (1971) proves that a game is convex if marginal contributions never decrease with coalition size: $S \subset T \;\Rightarrow\; MC_i^v(S) \leq MC_j^v(T)$. There is a clear parallel between this inequality and the nonnegativity of second order derivatives that characterizes convex functions: increasing returns to size is the characteristic of convexity. The converse is actually true (Ichiishi, 1981).

Two players *i* and *j* are *substitutable* if they contribute identically for all coalitions containing them: $MC_i^v(S) = MC_j^v(S)$ for all *S* containing *i* and *j*. A player *i* is *null* if he never contributes: $MC_i^v(S) = 0$ for all $S \subset N$.

Let Π_N denote the set of all possible players' orderings. The *marginal contributions vector* $\mu^\pi(N,v)$ associated to the ordering $\pi = (i_1, i_2, \ldots, i_n) \in \Pi_N$ is defined by:

$$\begin{aligned} &\mu^\pi_{i_1}(N,v) = v(i_1), \\ &\mu^\pi_{i_k}(N,v) = v(i_1, \ldots, i_k) - v(i_1, \ldots, i_{k-1}), \;\; k = 2, \ldots, n. \end{aligned} \tag{2}$$

In any given ordering, players obtain their marginal contribution depending on their position in the ordering. There are *n*! marginal contribution vectors, not necessarily distinct, and each vector is an allocation: $\mu^\pi(N,v) \in X(N,v)$. A convex game is strictly convex *if and only if* its marginal contribution vectors are distinct.

Given a set of players *N*, the set $G(N)$ of all characteristic functions on *N* coincide with the Euclidian space $\mathbb{R}^{2^n - 1}$. In his proof of the uniqueness of the value, Shapley (1951, 1953) uses the fact that the collection of *unanimity games* $(N, u_T)_{T \in \mathcal{C}(N)}$ defined by:

[4] Convexity (also called supermodularity) of transferable utility games has been introduced by Shapley (1971).

$$u_T(S) = 1 \quad \text{if } T \subset S,$$
$$= 0 \quad \text{if } T \not\subset S,$$

is a basis of the vector space $G(N)$: for any given function $v \in G(N)$, there exists a unique $(2^n - 1)$-dimensional vector $\alpha = (\alpha_T)_{T \in \mathcal{C}(N)}$ such that:

$$v(S) = \sum_{T \in \mathcal{C}(S)} \alpha_T \, u_T(S) = \sum_{T \in \mathcal{C}(S)} \alpha_T \;\Rightarrow\; v(N) = \sum_{T \in \mathcal{C}(N)} \alpha_T. \tag{3}$$

The α_T can be defined recursively, starting with $\alpha_\varnothing = 0$, as follows:

$$\alpha_T = v(T) - \sum_{S \in \mathcal{C}(T),\, S \neq T} \alpha_S \quad \Rightarrow \quad \alpha_T = \sum_{S \in \mathcal{C}(T)} (-1)^{t-s} v(S) \;\text{ for all } T \subset N.$$

The α_T are known as the *Harsanyi dividends*: α_T is the dividend accruing to coalition T once all sub-coalitions have obtained their dividends. *Positive games* are games whose dividends are nonnegative. Because unanimity games are convex, positive games are convex, as nonnegative linear combinations of convex games.

3. Solution concepts

Cooperative game theory offers a large variety of solution concepts, either as *subsets of allocations* $B(N,v) \subset X(N,v)$ meeting a number of restrictions, or as *allocation rules* $\varphi : G(N) \to X(N)$ that associate a particular allocation to a given game subject to some requirements. Notice that efficiency is included in the definition of an allocation rule.

Consider a game (N,v) and an allocation $x \in X(N,v)$. A minimal restriction is that each player should be allocated at least his individual worth: $x_i \geq v(i)$ for all $i \in N$. This defines *individually rational* allocations, known as *imputations*. The *core* extends these inequalities from individual players to coalitions:

$$C(N,v) = \left\{ x \in \mathbb{R}^n \;\middle|\; x(N) = v(N) \text{ and } x(S) \geq v(S) \text{ for all } S \subset N \right\}. \tag{4}$$

Each coalition receives at least its worth.[5] This is a requirement of *stability* in the sense that a coalition could raise an objection against an allocation if it was allocated less than its worth. In general, the core is a convex polyhedron of dimension at most n–1, possibly empty. Convex games have a nonempty (and typically large) core whose vertices are precisely the marginal contribution vectors defined in (2), as shown by Shapley (1971). It is easily verified that core allocations defined by (4) satisfy the complementary inequalities requiring that the amount allocated to a coalition should not exceed its contribution to the grand coalition:

$$x(N) \leq v(N) - v(N \setminus S) \quad \text{for all} \quad S \subset N. \tag{5}$$

In particular, the amount assigned to a player should not exceed his (marginal) contribution to the grand coalition: $x_i \leq v(N) - v(N \setminus i)$ for all $i \in N$.

Remark 1 When considering games with 2 or 3 players, the core is the set of allocations satisfying the inequalities $v(i) \leq x_i \leq v(N) - v(N \setminus i)$ for all $i \in N$. While these inequalities apply to core allocations of games involving any number of players, they do *not* alone characterize the core of games involving more that three players.

The core does not entail any value judgment: nothing ensures the fairness of core allocations. The core actually tends to capture the competitive forces that possibly exist within a game by favoring dominant players.[6] However,

[5] The core was introduced by Gillies (1953) in connection with the notion of stable set. It was later introduced as a solution concept by Shapley (1955). See Zhao (2018) for a clarification of the respective contributions of Gillies and Shapley as to the core.

[6] For example, in a 3-player voting game, if a player has a veto right, there is a unique core allocation where that player gets all. The glove market introduced by Shapley (1955) is another example.

when nonempty, the core contains allocations that treat substitutable players equally. Furthermore, the core allocates zero to null players. These observations lead to the concept of a *symmetric core* that only retains the core allocations that treat substitutable players equally.
Equal distribution of the surplus defined by:

$$EDS_i(N,v) = v(i) + \frac{1}{n}\left(v(N) - \sum_{j\in N} v(j) \right) \quad (i = 1,\dots,n)$$

is the simplest allocation rule. It coincides with the *equal division* in the case where $v(i) = 0$ for all $i \in N$. Assuming superadditivity, it defines an imputation that, however, does not take into account differences in contribution.

The *Shapley value* is an allocation rule that is *uniquely* defined by a set of axioms (Shapley, 1953). It satisfies two basic axioms that any allocation rule should satisfy: *symmetry* (substitutable players should be remunerated equally) and null *player* (players that do not affect coalitions' worth should not be remunerated), two requirements that are satisfied by the symmetric core. Remarkably, among all the rules that meet these two requirements, only one satisfies *additivity*: given two games on a common set of players, applying the rule to the game-sum is equivalent to applying the rule to the individual games: $\varphi(N,v_1) + \varphi(N,v_2) = \varphi(N,v_1+v_2)$. Shapley indeed proves that additivity, combined with efficiency, symmetry and the null player property, defines a unique allocation rule, namely the Shapley value.

Alternative axiomatizations have been proposed, two of which are quite remarkable. Young (1985) replaces additivity by *monotonicity*. Given two games on a common set of players, if the marginal contributions of a player are greater or equal in the first game than in the second, his remuneration should not be smaller in the first:

$$v_1(S) - v_1(S\setminus i) \geq v_2(S) - v_2(S\setminus i) \;\Rightarrow\; \varphi_i(N,v_1) \geq \varphi_i(N,v_2).$$

Myerson (1980) instead requires that players should gain (or lose) the same amount from cooperating:

$$\varphi_i(N,v) - \varphi_i(N\setminus j, v_{N\setminus j}) = \varphi_j(N,v) - \varphi_j(N\setminus i, v_{N\setminus i})$$

i.e. "*what I lose if you leave the game should be equal to what you lose if I leave the game*."

Hence, two major principles underlie the Shapley value: *contributions* and *fairness*. What a player obtains is exclusively related to his contributions and players that contribute identically are remunerated equally.

Remark 2 For a given player set *N*, these axiomatizations apply to the set *G*(*N*) of *all* games. They remain valid when restricted to the subset *SG*(*N*) of superadditive games.

There are several formulations of the Shapley value. One is based on the average marginal contribution vectors, assuming that orderings are equally likely:

$$SV_i(N,v) = \frac{1}{n!}\sum_{\pi\in\Pi_N} \mu_i^{\pi}(N,v) \quad (i = 1,\dots,n). \tag{6}$$

A second one is directly based on marginal contributions. Each player is allocated a weighted average of *his* marginal contributions:[7]

$$SV_i(N,v) = \sum_{S\in\mathcal{C}(N)} \alpha_n(s) MC_i(N,v) \quad (i = 1,\dots,n), \tag{7}$$

where the weights depend on coalition size and are given by $\alpha_n(s) = (s-1)!(n-s)!/n!$. A third one is based on Harsanyi dividends. By (3), we know that the sum of all dividends is equal to $v(N)$. An allocation can then be

[7] Formally the sum is limited to the coalitions containing player *i* although it makes no difference. Indeed, if a player is not a member of a coalition, his marginal contribution is null.

obtained by distributing the dividends of each coalition among its members. Harsanyi shows that the Shapley value allocates to each player the sum of the *per capita* dividend of the coalitions of which he is a member:

$$SV_i(N,v) = \sum_{T\in \mathcal{C}_i(N)} \frac{1}{t}\alpha_T \quad (i = 1,\dots,n). \tag{8}$$

The three definitions (6), (7) and (8) are equivalent.

Nothing ensures that the allocation defined by the Shapley value belongs to the core: it is a fair allocation against which coalitions may raise objections. However, when applied to convex games, the Shapley value defines an allocation that belongs to the symmetric core.

The *nucleolus* is another allocation rule that is always well defined (Schmeidler, 1969). It is a core allocation when applied to games whose core is nonempty, i.e. the nucleolus is a *core selection*. As an allocation rule, it satisfies the symmetry and null player axioms but fails to satisfy additivity, monotonicity and balanced contributions, all axioms that characterize the Shapley value. To quote Maschler, Peleg and Shapley (1979, p.303), the nucleolus results "*of an arbitrator's desire to minimize the dissatisfaction of the most dissatisfied coalition.*"[8] When the core is nonempty, this quotation could be reversed by saying that the nucleolus is "*the result of an arbitrator's desire to maximize the satisfaction of the least satisfied coalition*." Except for particular classes of games, there is no formula for computing the nucleolus. Another core selection is the *core center* that corresponds to its center of gravity introduced by Gonzáles-Díaz and Sánchez-Rodríguez (2007).

Remark 3 In the case of a convex game, the Shapley value, like the nucleolus and the core center, defines an allocation that is *centrally located* within the core. They all coincide when applied to games that are both convex *and* 1-convex, games whose core is a regular simplex (Driessen, 1985). The coincidence also happens for PS-games where the sum of the marginal contribution of a player to a coalition and its complement is a player-specific constant (Kar et al., 2009).

4. The n-Helix game

We denote by $N = \{1,\dots,n\}$ the set of organizations and by E_i the set of realizations of organization *i*. The set of all realizations is given by $E = \bigcup_{i\in N} E_i$. Realizations are not necessarily valued identically. The value of any set *A* of realization is known and given by $\Phi(A)$. We assume that the valuation function Φ is (finitely) *additive*:

$$\Phi(\varnothing) = 0 \text{ and } \Phi(A) = \sum_{e\in A}\Phi(e).$$

In case realizations are equally valued, the value of a set *A* of realizations is simply given by its cardinal |A|.

The problem we are addressing is the following: how to allocate the value of the set of all realizations $\Phi(E)$ between the organizations in a fair way, while avoiding objections from individual organization or coalition of organizations? To answer that question, we construct a transferable utility game (N,v) whose characteristic function *v* associates to each coalition *S* the value of the realizations made *exclusively* by the members of *S*:

$$v(S) = \Phi\left(\bigcup_{i\in S} E_i \setminus \bigcup_{i\in N\setminus S} E_i\right).$$

We then have $v(N) = \Phi(E)$ and the question concerns the allocation of $v(N)$, the total value of the realizations made by the *n* organizations.

[8] This procedure is to be compared to the leximin procedure introduced by Rawls (1971) that consists in making the worst-off player as well as possible.

The game (N,v) is the n-Helix game introduced by Mêgnigbêto (2018) for the case n = 3. Its definition can be simplified by associating to any subset of organizations the set of realizations its members have *in common and are the only ones to report*:

$$S \subset N \to \Omega(S) = \bigcap_{i\in S} E_i \setminus \bigcup_{i\in N\setminus S} E_i$$

with $\Omega(\varnothing)=0$. The family of 2^n-1 subsets $\{\Omega(S) \mid S \subset N, S \neq \varnothing\}$ forms a partition of the set E and the characteristic function can then be written as:

$$v(S) = \Phi\left(\bigcup_{T\subset S}\Omega(T)\right) = \sum_{T\subset S}\Phi\left(\Omega(T)\right) \text{ for all } S \subset N. \quad (9)$$

In the particular case where only the number of publications counts, we simply have:

$$v(S) = \left|\bigcup_{T\subset S}\Omega(T))\right| = \sum_{T\subset S}\left|\Omega(T)\right| \quad \text{for all } S \subset N.$$

Comparing (3) and (9), we observe that $\Phi(\Omega(T))$ is nothing but the Harsanyi dividend associated to coalition T. Dividends being non-negative, the Helix games are *positive* and thereby *convex*.[9] In a convex game, the incentives to join a coalition do not decrease as coalition grows. This is in line with the situations described by Helix games where complementarities play a central role. This is the *bandwagon effect* to which Shapley alludes in his 1971 paper.

Strict convexity arises if (though not only if) there are exclusive joint realizations among all coalitions involving at least two players and at most n–1: $\Omega(T) \neq \varnothing$ for all $S \subset N$ such that $2 \leq s \leq n-1$. Referring to the definition of strict convexity, we have:

$$\begin{aligned}\Delta(S_1,S_2) &= \sum_{T\subset S_1\cup S_2}\Phi\left(\Omega(T)\right) + \sum_{T\subset S_1\cap S_2}\Phi\left(\Omega(T)\right) - \sum_{T\subset S_1}\Phi\left(\Omega(T)\right) - \sum_{T\subset S_2}\Phi\left(\Omega(T)\right)\\ &= \sum_{\substack{T\subset S_1\cup S_2\\ T\not\subset S_1,S_2}}\Phi\left(\Omega(T)\right) \quad \text{for all } S_1,S_2 \subset N.\end{aligned}$$

We first observe that there is no coalition of size 1 in the last summation. Assuming that $S_1 \not\subset S_2$ and $S_2 \not\subset S_1$, we have $\Delta(S_1,S_2) > 0$ and, in the case where $S_1 \cup S_2 = N$, $\Delta(S_1,S_2) > \Omega(\Phi(N))$. Hence, strict convexity holds if $\Omega(i) = \varnothing$ for some i or $\Omega(N) = \varnothing$.

Remark 4 Positive games form an interesting class of convex games on which solution concepts tend to converge: the core coincides with the set of weighted Shapley values and with the Harsanyi set, the set of all distributions of dividends.[10] An axiomatization of the core on the set of positive games is given in Dehez (2024).

Using (8), the Shapley value is given by:

$$SV_i(N,v) = \sum_{T\in \Omega_i(N)} \frac{1}{t}\Phi\left(\Omega(T)\right)$$

and, to evaluate the share of each organization, we can use the normalized game $(N,\overline{v})$ defined by:

$$\overline{v}(S) = \frac{100}{v(N)}v(S).$$

9 Convexity of the 3-player Helix game has been investigated by Mêgnigbêto (2024a).

10 See Dehez (2017) for a synthesis of the relations between these solution concepts.

It defines a game that is *strategically equivalent* to the original game. The core and Shapley value being *covariant* solutions, this normalization does not affect these solution concepts in the sense that they follow the normalization. Hence, we have $SV_i(N,\bar{v}) = 100\, SV_i(N,v)/v(N)$, $i = 1,\dots,n$.

By convexity, the Shapley value defines a core allocation: following (5), every organization or coalition of organizations obtains *at least* the value of the realizations it has made *exclusively*, and *at most* its contribution to the grand coalition:

$$\sum_{T\subset S}\Phi\big(\Omega(T)\big) \le \sum_{i\in S}\sum_{T\in \mathcal{C}_i(N)} \frac{1}{t}\,\Phi\big(\Omega(T)\big) \le \sum_{\substack{T\subset N\\ T\not\subset N\setminus S}} \Phi\big(\Omega(T)\big) \quad \text{for all } S\subset N.$$

The volume of the core of a Helix game based on counting, relative to the volume of the imputation set, provides a measure of the extent of the synergies that exist between the participating organizations. The absence of synergies is a situation where the individual sets $E_1,\dots,E_n$ are disjoint. The core is then reduced to the single allocation x where $x_i = \Phi(E_i)$ $i = 1,\dots,n,$ and the synergy is null. The Shapley value, the nucleolus and the core center all coincide with that single allocation. At the other extreme, synergies are maximal if the individual sets are equal, $E_1 = E_2 = \dots = E_n = E.$ The resulting game is the unanimity game (N, u_N) and the core coincides with the imputation set. The Shapley value, the nucleolus and the core center all coincide with the equal division: $x_i = \Phi(E)/n$ $i = 1,\dots n.$

5. An example

Consider a situation involving four organizations and a set $E = \{a,b,c,d,e,f,g,h,p,q,r\}$ that includes eleven realizations. Individual realization sets are assumed to be given by:

$$E_1 = \{a,b,c,d\},\ E_2 = \{b,c,e,f,g,h\},\ E_3 = \{c,g,h,p,q,r\},\ E_4 = \{a,b,c,e,p,q,r\}.$$

The *nonempty* elements of the partition of the realization set E are then given by:

$$\begin{aligned}
&\Omega(1) = \{d\},\ \ \Omega(2) = \{f\},\\
&\Omega(1,4) = \{a\},\ \ \Omega(2,3) = \{g,h\},\ \ \Omega(2,4) = \{e\},\ \ \Omega(3,4) = \{p,q,r\},\\
&\Omega(1,2,4) = \{b\},\\
&\Omega(1,2,3,4) = \{c\}.
\end{aligned}$$

As there are empty elements of size larger than 1 in the partition, the resulting game is not strictly convex. If only the number of realizations enter into account, the associated game and its dividends are given by:

$$\begin{aligned}
v &= (1,1,0,0\,|\,2,1,2,3,2,3\,|\,4,5,5,7\,|\,11),\\
\alpha &= (1,1,0,0\,|\,0,0,1,2,1,3\,|\,0,1,0,0\,|\,1).
\end{aligned}$$

Then, using (8), the Shapley value is given by:

$$SV(N,v) = \left(\frac{25}{12},\frac{37}{12},\frac{11}{4},\frac{37}{12}\right);\ \ (2.08,\ 3.08,\ 2.75,\ 3.08).$$

The resulting ranking is $2 : 4 \text{ f } 3 \text{ f } 1$ and, referring to the normalized version, the Shapley value allocates the following shares (19, 28, 25, 28) in %. The nucleolus is given by (2.5, 3.25, 2.625, 2.625). It gives an almost similar ranking: $2 \text{ f } 3 : 4 \text{ f } 1.$ The game is not strictly convex: the core has 17 vertices. Its size is 47.87% of the size of the imputation set.[11] If the values of the realizations are given by (1, 2, 2, 3, 1, 2, 3, 3, 1, 2, 3), the associated game and its dividends are given by:

$$v = (3,2,0,0 \mid 5,3,4,8,3,6 \mid 11,9,10,15 \mid 23),$$
$$\alpha = (3,2,0,0 \mid 0,0,1,6,1,6 \mid 0,2,0,0 \mid 2).$$

Using (8), its Shapley value is given by:

$$SV(N,v) = \left(\frac{14}{3}, \frac{20}{3}, \frac{13}{2}, \frac{31}{6}\right); \ (4.67, 6.67, 6.50, 5.17).$$

The Shapley value allocates the following shares (42.4, 60.6, 59.0, 47.0) in %. and the resulting ranking is $2 \succ 3 \succ 4 \succ 1$. The nucleolus is given by (5.5, 6.75, 6.0, 4.75), revealing a different ranking: $2 \succ 3 \succ 1 \succ 4$.

6. Concluding remarks

The objective was to model the contributions to research and innovation of a group of organizations as a *n*-player transferable utility game, allowing for a differentiation of their contributions. We show that the resulting game has nonnegative Harsanyi dividends, implying the properties of convexity, superadditivity and monotonicity. Two measurements are considered: the relative size of the core to measure the extent of the existing synergies and the Shapley value to measure the contribution to research and innovation of each participant. We have privileged the Shapley value over other allocation rules, like the nucleolus or the core center, because it is based on a set of appealing properties. In particular, what a player obtains under the Shapley value is exclusively related to his contributions.

Several applications covering publications by the University-Industry-Government trio have been proposed. Mêgnigbêto (2024a) uses the data collected by Leydesdorff (2003) from the *Science Citation Index* 2000 to compute the Shapley value and the nucleolus, both resulting in the same ordering, namely $U \succ G \succ I$. There, the size of the core is shown to depend on the existence and importance of joint publications. Mêgnigbêto (2018) uses the *Web of Sciences* data over the period 2001-2010 in West Africa and in South Korea, showing that the same ordering applies. Mêgnigbêto (2024b) uses the same data, disaggregated in two sectors, domestic and foreign, showing that the foreign sector plays a more important role in West Africa than in South Korea.

In these analyses, the contributions were not differentiated. Collecting and organizing data is already not easy and differentiating the contributions increases the task significantly. When considering publications, one could use, for example, the *Impact Factor* which measures the relative influence of a journal.

The cooperative game defined in (9) can be applied to a large variety of situations, beyond the synergies in research and innovation. It can be for instance used for allocating damage caused by several tortfeasors, some damage being caused by several tortfeasors acting together. It has been applied to correctly assess the implication of companies involved in an antitrust investigation (Dehez & Ferey, 2024).

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Conflict of interest

All the authors claim that the manuscript is completely original. The authors also declare no conflict of interest.

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