Welfare fragmented information effects: The cost-benefit analysis and Trade-offs

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ABSTRACT

We offer an extensive analysis of the significance of information within the realm of Gaussian quadratic economies. We build upon the seminal papers of Morris and Shin (2002, 2007) and consider a signal game of incomplete information. Particularly, we question the suitability of partial transparency portrayed by fragmented information in addition to the private signal in terms of welfare effects. We can summarize our findings in two main points. First, fragmented information, in conjunction with a private signal, can reduce the reliance on public signals. Second, a conflicting effect arises between increasing full disclosure and increasing the precision of fragmented (semi-public) information when examining different complex scenarios, involving for example endogenous private information or imperfect correlated signals. For a critical threshold, an optimal communication strategy designed by fragmented information should be implemented whenever that kind of information is acquired at a high precision.

KEYWORDS

Transparency; Semi-public information; Private information; Trade-off; Coordination games; Signal game with incomplete information

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1. Introduction

Information disclosure and transparency strategies are the core of any central bank's activities (see Carpenter, 2004; De Haan et al., 2007). Although earlier research established the effectiveness of more explicit and future-oriented communication practices (refer to Section 2 for details), additional public information disclosure is suffused with uncertainty (see Chortareas and Stasavage, 2002; Posen, 2003; Mishkin, 2004; Cukierman, 2009).

The Chief Economist of the European Central Bank Issing has pointed out an extensive research program on how to design an optimal communication and transparency strategy (2005, p. 72): ‘Striking the balance between the need for clear and simple messages and the need to adequately convey complexity is a constant challenge for central bank communication’. Issing (2005, p. 72) made the example of the ECB which "...decided to publish neither the minutes of the Governing Council nor information about the voting behavior of its members, but instead holds an extensive monthly press conference directly after the council meetings." Lowering disclosure by the ECB "casts doubt on its determination to be transparent and accountable."

Scientific research recognized the merits of such claims, some of which used a game theoretic approach. Seminal papers by Morris and Shin (2002) and Amato et al. (2002) sparked a debate on the value of transparency through a static coordination game with imperfect common knowledge and incomplete information. Economic agents possess private and public information about the unknown state of economic fundamentals. Regarding this framework, private information is interpreted as insider information or, simply as, a personal interpretation of commonly accessible information. It can represent any information that an individual has observed, such as news received through private discussions (Stasavage, 2002). That type of signal differentiates potentially within the market participants. The second type of signal is public, which is commonly shared by all agents. The public signal can represent information gleaned from newspaper articles or other sources that report on central bank procedures (Stasavage, 2002). Public information plays a dual role: it conveys information about the fundamentals and acts as an anchoring point for coordination. The coordinating device is likely to push agents to underestimate their private signals, potentially leading to a social loss (Morris and Shin, 2002). Thereby, partial transparency is likely to engender better outcomes. We, particularly, investigate the welfare effects of fragmented information in the presence of private signals. Within this framework, public information is common among a subset (1/n of the population) of agents. Thus, fragmented information is public within the same sub-group of agents but is potentially different and unknown between all sub-groups of agents. Since this type of signal combines public and private elements, it is referred to as semi-public.

This paper proposes strategies for central banks to manage information dissemination within monetary policy frameworks. It suggests that when public announcements could harm welfare, introducing some ambiguity in their interpretation can reduce their impact and improve outcomes. This challenges the traditional view that central banks should either disclose all information or remain silent. By using fragmented public information, it is possible to avoid extreme outcomes and enhance the credibility of policy results.

- First, we show that fragmented public information (called semi-public information) combined with a private signal reduces private agents’ overreaction to public information.
- Second, we determine the conditions under which a trade-off occurs when the central bank faces the option of releasing noisier and full public information and the option of disclosing n semi-public information with high precision. We establish different classes of payoff functions and a generalization of the types of signals.
- Third, we challenge some theoretical insights from an experimental point of view.

Theoretical and experimental deliberations extract these observations: The (potential) conflicting effects of information on welfare depend on the payoff function and presumptions about the information structure.

The novelty of the paper stems from suggesting different scenarios in which central banks are recommended
to follow a partial transparency strategy in the form of semi-public information, which is common within a group of agents but different and unknown for the remaining groups, along with specific information for each agent (private information) whether this information is acquired at a cost or not. All agents still benefit from public information but don’t share the same between all sub-groups of agents.

Agents react to the two types of information aspects and make their choices using Bayesian inference about the underlying state of the economy. In this sense, we allow variations in the payoff function (welfare loss) and study how this function is governed by the parameters of interest, including the precision of the disclosed signals, the number of semi-public signals (the fragmentation measure), and the coordination motive. Particularly, we distinguish three cases: Case 1 pertains to “Actions get right”. It specifies a situation where agents care only about being close to the unknown state of fundamentals. The second case pertains to “reducing heterogeneity”. It characterizes agents who benefit from being close to each other whatever the economic fundamental is. Case 3 mixes case 1 and case 2.

In addition, we dissect how the welfare effects of information depend on whether they are provided exogenously or at a certain price. For instance, that assumption was only discussed when full public information disclosure holds. Importantly, we oppose situations in which strategic complementarities can generate positive/negative externalities to a situation with no direct externalities. Positive externalities arise when agents have incentives to be close to one another independently of the consensus action (Lindner, 2007).

These supplementary constraints permit to develop of a richer game theoretic framework and to upgrade previous literature, many of which we review in Section 2, focused on singular aspects.

The outline of the paper is given as follows. Section 2 examines the "good" and "dark" sides of full public information disclosure in literature. In Section 3, we establish a short-stylized model of the reception of two types of signals. In Section 4, we disentangle the welfare effects under various assumptions. Section 5 offers experimental insights. We conclude in Section 6.

2. Benefits and costs of public information in theory and practice

Additional public disclosure raised concerns among central bankers. Proponents of transparency claimed that openness coordinates expectations on the central bank’s price stability objective. According to Morris and Shin (2002), public information carries mixed purposes. On the one hand, it conveys information about fundamentals. On the other, it acts as a coordinating device for the agents’ beliefs, possibly contributing to lower welfare (Amato and Shin, 2003). The overall Morris and Shin’s analysis is therefore contingent on a socially harmful coordination (Angeletos and Pavan, 2007). After calibrating some of the theoretic game’s parameters, Svensson (2006) found that public information is always welfare-enhancing since public information is naturally more precise than private information. Replying to Svensson (2006), Morris et al. (2006) showed that if the public and private signals are correlated, more precise public information can reduce welfare even if public information is released at high accuracy.

Morris and Shin (2005) assumed endogenous public information and ascertained the (potential) negative effects of public information. Regardless of the model’s parameters, James and Lawler (2011) agreed with Morris and Shin (2002) that more transparency necessarily decreases welfare whenever the payoff function accounts for the central bank’s policy intervention. Woodford (2005) argued that the damaging effect of public information is generated as the coordination motive (called "beauty contest" term) is collapsed at the aggregate level of welfare. Demertzis and Hoeberichts (2007) reinforced the argument of Morris and Shin (2002) when private information is provided at a cost, while overreaction to public information depends on its precision according to Dale et al. (2008). Whenever private information is acquired at a cost, a trade-off emerges between information acquisition and
information (in)efficient use of information (see Myatt and Wallace, 2015; 2018; 2019). Baeriswyl (2018) considered rather endogenous public information and noted that available private information does not shift the optimal degree of public information accuracy. The nature of strategic environment is relevant though. Arato et al. (2021) considered public information acquisition in the presence of partial announcement à la Cornand and Heinemann (2008). Such a strategy surely mitigates the overreaction to full public disclosure but the associated price should be contingent on the fraction of informed agents. James and Lawler (2012a, 2012b) augmented the model of James and Lawler (2011) by allowing for heterogeneous precisions of private information. Accounting for the central bank’s policy in the form of a linear commitment rule cheers opacity as the optimal strategy. However, there is a strong case for partial transparency whenever agents exploit their private information to update their beliefs (James and Lawler, 2017). A class of abstract games included a sender-receiver game where the sender’s profit attains its maximum under a partial disclosure strategy (Rayo and Segal, 2010).

More public releases to steer the market’s forecasts are not always welcome if they don’t clearly instruct about financial markets and prices (see Connolly and Kohler, 2004; Andolfatto, 2010). According to Muller and Zelmer (1999), extra information in monetary policy reports has facilitated price adjustments in interest rates and exchange rates. Jensen (2002) observed negative consequences of transparency when central banks’ preferences are more inclined towards low inflation objectives than output gap stabilization. Both objectives should be viewed as complements rather than substitutes (Thornton, 2003). Within the same line of thoughts, Baeriswyl and Cornand (2018) argued a monetary policy that prioritizes output-gap stabilization, even when the central bank’s operations are not entirely transparent, can be optimal and effective in addressing cost-push shocks. Antal et al. (2004) argued that while transparency has benefits over fundamental economics, it might lead to strategic uncertainty and too inefficient outcomes. Once ambiguity and noisiness are accounted for in the disclosed signals, increased transparency might reduce financial instability (Heinemann and Illing, 2002). Amador and Weil (2010) showed that increasing the precision of public information may increase the uncertainty about monetary shocks. The authors recommended releasing either all of the information or none of it. Sánchez (2013) emphasized the relevance of central bank knowledge in assessing the impact of public information disclosure on macroeconomic stability in a setup where private agents react to ambiguity. The simulation results cheer higher transparency for better welfare particularly when there is a high level of central bank awareness regarding policy weight. Within a specific literature of macroeconomic models, Hellwig (2005) and Roca (2010) claimed that heightened transparency consistently enhances welfare in New Keynesian models driven by nominal demand or supply shocks. Candian (2021) corroborated those observations within complete assets markets. Otherwise, public information exhibits demand imbalances. Nevertheless, Brzoza-Brzezina and Kot (2008) argued that public information is of little importance if macroeconomic projections are provided. Walsh (2007) and Goesselin et al. (2009) challenged the idea of a transparency threshold depending on central banks’ expectations of demand and supply shocks. Addressing the question of business cycles matters when assessing the social value of public information (Angeletos et al., 2016). Firms’ decisions are akin to monetary policy state and conduct. Within real rigidity, more information enhances welfare when the cycle is propelled by beneficial forces like technology shocks. Conversely, it diminishes welfare when driven by detrimental forces like markup shocks. Additionally, information acquisition technology plays a vital role in determining the optimal level of public disclosure. Such an argument was challenged by Chahrour et al. (2014) and Goldstein and Yang (2017). Public information could be socially costly because it inhibits the transmission of private information. This is likely to occur when market participants are nearly risk-neutral, resulting in forecast errors (Kool et al., 2011). Middeldorp (2010) and Middeldorp and Rosenkranz (2011) evidenced that the subjects tend to invest more in public information if provided at no cost. The experiment by Dale and Morgan (2012) assessed the theoretical argument of Morris and Shin (2002). The authors noted a decline in the aggregate welfare stemming from inefficient information utilization and heightened unpredictability in
decision-making. Crucially, individuals prioritize public information, despite its lower precision compared to private information.\textsuperscript{1} The disproportionate weight is incentivized by strategic complementarities.\textsuperscript{2} Cornand and Heinemann (2014a) challenged this idea by conducting experiments where the coordination motive varies through treatments and noted the respective weights put on the public signal. While the experimental result is not as pronounced as the theoretical one, the overall observations are aligned towards recommending a partial transparency strategy. Since agents’ actions are driven by Bayesian higher order-beliefs from a theoretical perspective but humans have limited cognitive capabilities, previous theoretical arguments should consider limited levels of reasoning (Cornand and Heinemann, 2015). In the same vein, Baeriswyl and Cornand (2014) established that a limited precision of public information such as proposed by Heinemann and Illing (2002) doesn’t seem to have an impact on agents’ behavior but is still recommended as a communication design from a practical point of view. Trabelsi and Hichri (2021) supported the observations of Baeriswyl and Cornand (2014) and testified to an overreaction to public information. They argued that while partial publicity works as an efficient tool for the crowding in of private information, concerns related to discrimination and fairness should be warranted since uninformed agents are underprivileged. Andolfatto et al. (2014) claimed that non-disclosure about future asset returns is desirable if participants do not access hidden information at a very low cost. Further evidence about the crowding out of private information is found in Binz et al. (2023). Central bank economic transparency induces managers to rely less on stock prices when making investment decisions.

The costs and benefits of central banks’ disclosure hinge on the degree of trustfulness and clarity of conveyed messages. For example, households are less sensitive to the European Central Bank (ECB)’s communication whenever they have doubts about its content (Baerg et al., 2018). Comparing investors’ behavior on days when a press conference takes place after the Federal Open Market Committee announcements to those where the Federal Reserve does not hold a press conference, reveals that investors’ coordinating attention induces a welfare loss if a press conference follows less precise announcements (Boguth et al, 2019). Further papers stressed the crowding out effect of private information induced by an overreaction to public information when it comes to considering consensus against individual forecasts. The latter attach more importance to public releases. The result was put forward by Bordalo et al. (2022), consolidating the views of Coibion and Gorodnichenko (2015). Yet, transparency overcomes inflationary bias under a discretionary policy and makes central banks more credible but still has ambiguous impacts on welfare (see Duffy and Heinemann, 2021). A learning-to-forecast experiment was proposed by Kryvtsov and Peterson (2021) to study the effect of central bank communications on individuals’ forecasts following monetary policy shocks. Others focused on the agents’ reaction to central bank disclosures in the context of financial stability such as Chakravarty et al. (2021).

There is an interesting number of studies that dealt with the subject from an empirical perspective (Andersson et al., 2006; Kohn and Sack, 2004; Ehrmaan and Fratzscher, 2007a; Trabelsi, 2016a; Trabelsi, 2016b). Ehrmann et Fratzscher (2007a) showed that communication about the Federal Open Market Committee’s decisions lowers the short-term predictability of macroeconomic variables, supporting partially the argument of Morris and Shin (2002). van der Cruijsen et al. (2010) provided evidence of a critical state, below which increasing transparency (measured for instance by an index) reduces inflation persistence, and above which more transparency has the opposite effect. A more recent strand of literature uses panel regressions and data from professional forecasters, including Trabelsi (2016b), Lustenberg and Rossi (2018), Rai et al. (2023), and De Mendonça et al. (2023) showed that forecast errors and dispersion are sensitive to the degree of central bank’s communication. Zhang et al. (2023) focused on the relationship between central bank transparency and systemic risk. They depicted a non-linear effect with a

\textsuperscript{1} For a comprehensive survey of related experiments, we recommend Cornand and Heinemann (2014b, 2019).

\textsuperscript{2} Strategic complementarity refers to the situation where a change in one player’s choice positively affects the marginal payoff of the other player, as discussed in Eichberger and Kelsey (2002), or Potters and Suetens (2009), Cooper (1999). Conversely, strategies are considered non-complementary if a player cannot enhance her payoff when the other player alters her action.
moderating role of increased central bank independence, while higher transparency levels seemed to have a stabilizing effect on exchange rate volatility in a set of Asian emerging countries (Aftab and Mehmood, 2023).

The stream of literature extending Morris and Shin’s framework suggested alternative information structures with their respective (potentially improved) welfare implications:

(i) **Partial publicity**: Optimal transparency does not necessarily mean making public information fully available. The central bank can release the public signal only to a subset P of agents (Walsh, 2007; Cornand and Heinemann, 2008). While, Walsh (2007) considered the number of public information disclosed, Cornand and Heinemann (2008) defined transparency as containing two components: the degree of publicity and the accuracy of public information.

(ii) **Ambiguous public information**: Assuming a Lucas-Phelps Island economy, Myatt and Wallace (2014) proposed imperfectly correlated signals, known as announcements with "limited clarity". Arato and Nakamura (2011) argued that mixing private noise with public information might be welfare improving. Central banks should acquire information with high quality and make fewer clear announcements.

(iii) **Fragmented information**: Morris and Shin (2007) considered a different information structure. In addition to full public information, the central bank discloses n semi-public signals. Each signal is observed by 1/n of the set of economic agents. The signal is semi-public, which is common among agents who belong to the same group but potentially differ from one group to another. Fragmented information can be established through cheap talks used by central banks. Speeches by governors may be considered a fragmented way of communication. Common knowledge does not hold across agents. Thus, fragmentation is meant to be another form of "limited publicity".

The point (iii) is the purpose of our work. Before proceeding to our analysis, we note what follows:

1. Heightened precision of public information with "limited publicity" as suggested by Cornand and Heinemann (2008) is difficult to implement in practice unless excluding some fraction of information users is endogenously given in the model (see Arato and Nakamura, 2021). Kim (2010) extended the models of Morris and Shin (2002) and Cornand and Heinemann (2008) by supposing local exchange of private information between agents. The authors concluded that increasing transparency (in terms of accuracy and publicity) is always welfare-enhancing. ³

2. The results obtained by Cornand and Heinemann (2008) and Arato and Nakamura (2011) are akin to the loss function (which is the same as in Morris and Shin, 2002). Indeed, the welfare function used by Morris and Shin (2002) is controversial since the detrimental effect of transparency is driven by the relative relevance of coordination and stabilization at the social level.

Yet, these works miss important assumptions either about the payoff function or the informational structure. Particularly, we lay out theoretical settings including fragmented (semi-public) and private signals under different assumptions of information acquisition (costly private information or not), loss functions, and externalities (absence or positive externalities).

Our analysis heavily builds on Morris and Shin (2002, 2007), Lindner (2007), and Trabelsi (2013). While interesting, each of these works misses important constraints. Addressing the question of trade-off implies mixing fragmented and private information. Such a heterogeneous structure is close to the seminal paper of Morris and Shin (2002) and intertwines the welfare effects concerning:

- Quantity (the number of semi-public signals released).
- Quality (the precision of both types of information).

³ Kim (2010) considers the same loss function as in Morris and Shin (2002).
The coordination motive.

The privilege brought by endogenous private information will also refine the set of equilibrium and have implications on welfare loss. In either case, a conflicting effect emerges and central banks face a trade-off.

3. The setup

The model is a two-stage game in which the central bank decides about the degree of fragmentation (noted \( n \)) – the number of semi-public information – in the first stage, and agents acquire their private information in the second stage. There is a continuum of agents, indexed by the unit interval \([0,1]\). Agent \( j \) selects an action \( a_j \in \mathcal{R} \), and \( \bar{a} = \int a_j \, dj \) stands for the action profile over all agents. Agent \( j \)'s best response according to the Bayesian inference is given by

\[
a_j = (1 - r)E_j(\theta) + rE_j(\bar{a}) \tag{1}
\]

where \( E_j \) represents the agent \( j \)'s conditional expectation on her available information, \( \theta \) is the state of the economic fundamentals and \( r \in (0,1) \) measures the degree of strategic complementarities, called also the “beauty contest” term or the coordination motive. The optimal action for an individual \( j \) is a function of two components: the opinion about the state \( \theta \), and the expectation of average action formed by all individuals.

Agents minimize the same loss function ('Mixture', \( M \) henceforth)

\[
E(L^M) = (1 - r) \int (a_j - \theta)^2 \, dj + r \int (\bar{a}_j - a_j)^2 \, dh \, dj \tag{2}
\]

The term \( (a) \) is the loss arising from the individual forecast error ('Action get right', \( AR \) henceforth). The term \( (b) \) is the loss arising from disagreements across individuals on the estimate of \( \theta \) ('Reducing heterogeneity', \( RH \) henceforth)

Each group \( i=1, 2, \ldots, n \) receives the same semi-public signal from the central bank

\[
Z_i = \theta + \eta_i \quad \text{with} \quad \eta_i \sim N \left( 0, \frac{1}{\gamma} \right)
\]

Additionally, each agent \( j \) has a specific private signal that differs potentially across all individuals

\[
x_j = \theta + \epsilon_i \quad \text{with} \quad \epsilon_i \sim N \left( 0, \frac{1}{\beta} \right)
\]

Actions \( a_j \) are linear functions of both signals

\[
a_j = \lambda Z_i + (1 - \lambda)x_j \tag{3}
\]

Substituting \( E_j(\theta) \) and \( E_j(\bar{a}) \) by their expressions in Equation (1) and equating coefficients in Equation (1) and Equation (3), we can solve for \( \lambda \) as follows
\[ a_i^j = \frac{\gamma z_i + \beta (1 - \frac{1}{n}) x_j}{\gamma + \beta (1 - \frac{1}{n})} \]  (4)

where
\[ \lambda_{eq} = \frac{\gamma}{\gamma + \beta (1 - \frac{1}{n})} \]  (5)

Agents attribute a weight to the semi-public signal such that \( E(L_{PS}) \) is minimized (see Appendix B.1 for details).

We can calculate the average action across agents as follows
\[ \bar{a} = \int a_i d_j = \frac{\gamma z_i + \beta (1 - \frac{1}{n}) \theta}{\gamma + \beta (1 - \frac{1}{n})} \]  (6)

According to Equation (6), imprecise semi-public information (\( \gamma \rightarrow 0 \)), unlimited number of fragmented information (\( n \rightarrow \infty \)), or extremely precise private information (\( \beta \rightarrow 0 \)) disregard the coordinating role of the semi-public signal. Conversely, under very precise semi-public information (\( \gamma \rightarrow \infty \)) or imprecise private information (\( \beta \rightarrow \infty \)) semi-public information, coordinating behavior shows up (see Table 1).

**Table 1.** Sensitivity of the average action to the accuracy of semi-public and private information and the fragmentation measure.

<table>
<thead>
<tr>
<th>( \lim() )</th>
<th>( \gamma \rightarrow 0 )</th>
<th>( \gamma \rightarrow \infty )</th>
<th>( \beta \rightarrow 0 )</th>
<th>( \beta \rightarrow \infty )</th>
<th>( n \rightarrow \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{a} )</td>
<td>( \theta )</td>
<td>( \tilde{Z} )</td>
<td>( \tilde{Z} )</td>
<td>( \theta )</td>
<td>( \theta )</td>
</tr>
</tbody>
</table>

Bayesian higher-order beliefs justify why agents still append a greater weight on the semi-public signal (see Eq. (5)) since it surpasses the informational content of the same type of information, given by the relative precision \( \gamma / \gamma + \beta \).

As the coordination motive (\( r \)) gains importance, the agents’ average dispersion becomes maximal (\( \frac{\partial \lambda}{\partial r} > 0 \)). However, weight at equilibrium (\( \lambda_{eq} \)) degenerates whenever the fragmentation measure is high (\( \frac{\partial \lambda}{\partial n} < 0 \)).

Considering \( n=1 \) pertains to a fully public information disclosure such as described in Morris and Shin (2002). The unique equilibrium is given by
\[ a_j = \frac{\gamma z + \beta (1 - \frac{1}{n}) x_j}{\gamma + \beta (1 - \frac{1}{n})} \]  (7)

Yet, the weight put on the public signal in Equation (7) exceeds the relative precision \( \frac{\gamma}{\gamma + \beta} \) as well as \( \frac{\gamma}{\gamma + \beta (1 - \frac{1}{n})} \) when \( n \geq 2 \), mirroring a disproportionate effect of the (semi-) public signal on the social coordination.  

\[ \frac{\gamma}{\gamma + \beta} \leq \frac{\gamma}{\gamma + \beta (1 - \frac{1}{n})} \leq \frac{\gamma}{\gamma + \beta (1 - r)}. \]
4. Welfare effects, Trade-off, and policy implications

We establish transparency as having two items: How accurate the semi-public signal ($\gamma$) is and how many sub-groups are ($n$). We then investigate how these items alter agents' interests based on their objectives (see Table 1). We start by looking at how the overall welfare loss is affected by the coordination motive ($r$) in relation to average expectations. A higher degree of strategic complementarities ($r$) makes actions diverge from the optimal point because agents underestimate their private information. At the same time, the coordination motive bridges the gap between individual actions, which seems to be more influential: Increasing the level of strategic complementarity decreases the combined losses from AR and RH. For all cases (AR, RH, and M), improving the accuracy of semi-public information ($\gamma$) and private information ($\beta$) consistently reduces the private sector loss.

Table 2. The loss functions and their respective first derivatives.

<table>
<thead>
<tr>
<th>Case 1: Actions get right</th>
<th>Case 2: Reducing heterogeneity</th>
<th>Case 3: Mixture</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\int (a_j^i - \theta)^2 , di , dj$</td>
<td>$\int \int \int (a_k^j - a_j^i)^2 , di , dk , dh$</td>
<td>$(1-r) \int (a_j^i - \theta)^2 , di , dj$</td>
</tr>
<tr>
<td>$\frac{2\beta r}{n^2} \bigg[ \frac{y + \beta \left(1 - \frac{r}{n}\right)}{y + \beta \left(1 - \frac{r}{n}\right)^3} \bigg] \geq 0$</td>
<td>$\frac{4\beta r - 1}{n^2} \bigg[ \frac{y + \beta \left(1 - \frac{r}{n}\right)}{y + \beta \left(1 - \frac{r}{n}\right)^3} \bigg] \leq 0$</td>
<td>$-\frac{\gamma}{n} \bigg[ \frac{y + \beta \left(1 - \frac{r}{n}\right)}{y + \beta \left(1 - \frac{r}{n}\right)^3} \bigg] \leq 0$</td>
</tr>
<tr>
<td>$\frac{\beta \left(1 - \frac{r}{n}\right) \left(2 \frac{r}{n} - 1\right) - y}{y + \beta \left(1 - \frac{r}{n}\right)^3} \leq 0$</td>
<td>$2\beta \left(1 - \frac{r}{n}\right) \left(2 \frac{r}{n} - 1\right) - y \left(1 - \frac{1}{n}\right)$</td>
<td>$-\left(1 - \frac{r}{n}\right) \bigg[ \frac{y + \beta \left(1 - \frac{r}{n}\right)}{y + \beta \left(1 - \frac{r}{n}\right)^3} \bigg] \leq 0$</td>
</tr>
<tr>
<td>$\frac{-2\beta r^2}{n^3} \bigg[ \frac{y + \beta \left(1 - \frac{r}{n}\right)}{y + \beta \left(1 - \frac{r}{n}\right)^3} \bigg] \leq 0$</td>
<td>$2\beta y \frac{r}{n^2} \left(1 - r\right) + y^n \bigg[ \frac{y + \beta \left(1 - \frac{r}{n}\right)}{y + \beta \left(1 - \frac{r}{n}\right)^3} \bigg] \geq 0$</td>
<td>$\frac{\gamma r^2}{n^3} \bigg[ \frac{y + \beta \left(1 - \frac{r}{n}\right)}{y + \beta \left(1 - \frac{r}{n}\right)^3} \bigg] \geq 0$</td>
</tr>
<tr>
<td>$-\frac{y \left(1 - \frac{r^2}{n^2}\right) + \beta \left(1 - \frac{r}{n}\right)^3}{y + \beta \left(1 - \frac{r}{n}\right)^3} \leq 0$</td>
<td>$2 \left(1 - \frac{r}{n}\right) \frac{y \left(2 - r - 1\right) - \beta \left(1 - \frac{r}{n}\right)^2}{y + \beta \left(1 - \frac{r}{n}\right)^3} \leq 0$</td>
<td>$-\left(1 - \frac{r}{n}\right)^2 \bigg[ \frac{y + \beta \left(1 - \frac{r}{n}\right)}{y + \beta \left(1 - \frac{r}{n}\right)^3} \bigg] \leq 0$</td>
</tr>
</tbody>
</table>

Conclusion: Accurate fragmented policy is preferred

Notes: $\gamma$ and $\beta$ are the precisions of semi-public and private information, respectively. $n$ is the fragmentation measure.

Since $0 \leq r \leq 1$ and $n \geq 2$, we have $1 - \frac{r}{n} > 0$. 

Since 0 ≤ r ≤ 1 and n ≥ 2, we have 1 − r/n > 0.
4.1. Trade-off under different payoff assumptions

4.1.1. The case of 'Actions get right' (AR)

We explicit the overall welfare loss of the private sector (PS) provided in the first column of Table 1 (see proof in Appendix B.1)

\[ E(L_{AR}^{PS}) = \int (a_j - \theta)^2 \, dj = \frac{r + \beta(1 - \frac{r}{n})^2}{r + \beta(1 - \frac{r}{n})} \]  

(8)

From Eq. (8), the expected welfare loss eliminates the coordination motive and dovetails to the sum of quadratic dispersion to the fundamentals.

Providing highly accurate semi-public information necessarily drops the loss value only when at least two fragmented information are released (see Figure 1). The overall loss is nonlinear with respect to the precision of the semi-public signal. Now, a fragmented policy is always welfare-improving \( \left( \frac{\partial E(L_{AR}^{PS})}{\partial n} \leq 0 \right) \). It is noteworthy that these welfare effects are derived from the assumption of the payoff structure which is judgmental and subjective. A coordination loss is incurred by canceling the potential of semi-public signals between two groups.\(^5\)

![Figure 1](image)

**Figure 1.** Evolution of the welfare loss according to the semi-public signal’s accuracy: Case 1 'Actions get right'.

Parameters used for calibration \((r = 0.7, \beta = 1)\).

---

\(^5\) Mathematically, the social optimal action would be \( a_j^* = E_j(\theta) \) rather than the expression given by Equation (4).
4.1.2. The case of ‘Mixture’ (M)

The expected loss accounts for disparities in the view of fundamentals as well as disparities between individual actions (see Appendix B.1 for proof)

\[
E(L_M^s) = (1 - r) \int \int (a_j - \theta)^2 \, di \, dj + \frac{r}{2} \int \int (a_h^k - a_j^k)^2 \, di \, dj \, dk \, dh
\] (9)

We discern unclear effects on the expected loss in Equation (9) if parallel shifts in the precision of the semi-public signal and fragmentation measure \(n\) are introduced (see Figure 2). Contrary to the case of 'AR', the fragmentation measure has adverse effects on the social loss, while keeping the precision of the semi-public signal constant \(\left.\frac{\partial E(L_M^s)}{\partial n}\right| < 0\).³

\[\text{Figure 2. Evolution of the welfare loss function according to both transparency items. Parameters used for calibration (r = 0.5, } \beta = 1).\]

Next, we compare various scenarios with respect to welfare effects. The central bank is skeptical about less precise but fully available public information \(n=1\) and more accurate semi-public information (see Figure 3). Point A stands for the first scenario \(n=1, \gamma_A \text{ low}\) which generates the loss \(E(L_M^s)_A\). Conflicting effects from more precise and fragmented information follow:

- At Point B(\(\gamma_B > \gamma_A, n=2\)): \(E(L_M^s)_A < E(L_M^s)_B\): The central bank should make all information available.
- At Point C(\(\gamma_C > \gamma_A, n=1\)): \(E(L_M^s)_A > E(L_M^s)_C\): The optimal strategy boils down to fragmented information with high precision.
- Now, let D be another point that corresponds to precision \(\gamma_D\), a fragmentation measure \(n =2\), and a
corresponding loss $E(L^{PS}_M)_D$. At point C($\gamma < \gamma_D$, $n=1$): $E(L^{PS}_M)_C = E(L^{PS}_M)_D$: No clear-cut preference for either strategy.

PROPOSITION 1 According to the loss function (Case 3: Mixture), accurate fragmented public information outperforms other strategy choices if the ratio (high to low) precision is greater than $1 - \frac{r}{n(1 - r)}$.

![Graph](image_url)

**Figure 3.** Sensitivity of the overall loss to the fragmentation measure $n$. Case 3: Mixture. Parameters used for calibration ($r = 0.5, \beta = 1$).

4.1.3. The case of a loss function with positive externalities

In this section, we aim to analyze the equilibrium and welfare effects accounting for positive externalities. Lindner (2007) suggests that positive externalities occur when agents benefit from neighboring, regardless of the average dispersion. The expected welfare loss takes the following formula

$$E(L^{PS}_H)_pe = (1 - r) \iint (a_i^j - \theta)^2 \, di \, dj + r \iint \iint (a_i^h - a_i^j)^2 \, di \, dj \, dk \, dh \quad (10)$$

The optimal action of agent $j$ of a group $i$ has now this expression

$$a_i^j = \frac{1-r}{1+r} E_i^j(\theta) + \frac{2r}{1+r} E_i^j(\bar{\theta}) \quad (11)$$
We introduce formulas of $E_i^j(\theta)$ and $E_i^j(\hat{a})$ in Equation (11) and equalize coefficients in Equation (3) and Equation (11). We get

$$\lambda_{eq,pe} = \frac{\gamma(1+r)}{\gamma(1+r)+\beta(1+r-r^2n)}$$  \hspace{1cm} (12)

**Proposition 2** Under positive externalities, accurate fragmented information is the optimal choice if $$\frac{\gamma_2}{\gamma_1} \geq \frac{1+r-\frac{r^2}{n}}{1-r}.$$

4.1.4. The case of a loss function with negative externalities

It is interesting to investigate the welfare effects of information when assuming that negative externalities arise because agents have an interest in being close to the consensus forecast. Such a situation can be modeled through the following loss function

$$E(L^P_n) = (1-r) \int (a_j - \theta)^2 \, di \, dj + r \int (a_j - \hat{a})^2 \, di \, dj$$  \hspace{1cm} (13)

The optimal action, the weight assigned to the semi-public signal, as well as the average action are as provided by Equations (1) to (6), respectively. One can show that under this assumption, more precise fragmented information is always welfare-improving.

**Proposition 3** Under negative externalities, accurate fragmented information is the optimal choice.

4.2. Trade-offs under different information structure assumptions

4.2.1. Endogenous private information

Previous research has primarily focused on the potential negative impact of more precise public information on welfare. This literature often assumes fixed precision of private information. Colombo et al. (2014) introduced an endogenous acquisition of private information, linking inefficiencies in acquisition to inefficiencies in information utilization. Thus, we allow agents to adjust the precision of their private information in response to changes in the quality of semi-public information and the number of signals disclosed by the central bank. In this section, we modify the loss function to account for linear costs incurred by the private sector to enhance the quality of their signal.

$$C^{PS}(\beta) = c\beta, \quad c > 0$$

Agents’ minimization program implies

$$E(T^P_n) = E(L^P_n) + C^{PS}(\beta) = \frac{1}{1+\beta} + c\beta$$  \hspace{1cm} (14)

The first-order condition of Equation (13) is given by

$$\frac{\partial E(T^P_n)}{\partial \beta} = \frac{-1}{(1+\beta)^2} + c = 0 \Rightarrow \beta^* = \max \left(0, \frac{1}{\sqrt{c}} - \frac{\gamma}{1+\beta} \right)$$  \hspace{1cm} (15)

We inspect the second-order condition

---

6 We treat the case of nonlinear costs in Appendix D and show that substitutability between precisions of semi-public and private signals holds.
Taking the derivative of Equation (15) with respect to the precision of the semi-public signal and the fragmentation measure, we extract the following

\[
\begin{aligned}
\frac{\partial^2 E(\tau_{\theta}^*)}{\partial \beta^2} &> 0 \iff \frac{2(1-n)}{[\gamma + \beta (1-n)]^3} > 0 \\
(16)
\end{aligned}
\]

According to the expression of Equation (17), the quality of private information is decreasing in the accuracy of semi-public information. The ratio \(1/1 - r/n\) exceeds 1. Further, the crowding-out (in) effect depends on how strong (weak) the coordination motive (r) is.

**Proposition 4** The private signal’s optimal precision is decreasing in the precision of semi-public information but increasing in the fragmentation measure. Both signals are strategic substitutes.

There is a trade-off under endogenous private information. This fact is caught up with these observations:

- Under fragmented public disclosure \((n \geq 2)\), minimizing the welfare loss (resp. maximizing the welfare) implies that the private sector is urged to invest more in the private signal \((\beta \uparrow)\).
- Improving the quality of the \(n\) semi-public signals \((\gamma \uparrow)\) minimizes loss (resp. maximizes welfare) when the precision of private signals is reduced \((\beta \downarrow)\).

### 4.2.1. Imperfect correlated signals

This section introduces a more complex scenario to examine the trade-off between enhancing commonality and utilizing more precise but fragmented information. We depart from Myatt and Wallace (2012, 2014, 2015, 2018, 2019) and Arato and Nakamura (2011) and extend the underlying information structure as well as the payoff function. Particularly, the central bank reveals \(n\) ambiguous semi-public information\(^7\)

\[
Z_i^j = \theta + \eta_i^x + \alpha_i^j = Z_i + \alpha_i^j \quad i=1,2,..,n \quad \text{with} \quad \eta_i^x \sim N \left(0, \frac{1}{\gamma} \right) \quad \text{and} \quad \alpha_i^j \sim N \left(0, \frac{1}{\beta} \right)
\]

Private information is expressed as

\[
x_i^j = \theta + \eta^x + \epsilon_i^j = \chi + \epsilon_i^j \quad \text{with} \quad \eta^x \sim N \left(0, \frac{1}{u} \right) \quad \text{and} \quad \epsilon_i^j \sim N \left(0, \frac{1}{\delta} \right)
\]

We define \(\varphi_z = \frac{\gamma \delta}{\gamma + \delta}^2\) as \(Z\)'s signal precision and \(\varphi_x = \frac{\beta u}{\beta + u}\) as the \(x\)'s signal precision. The term \(\delta\) stems from the correlation of idiosyncratic noise of semi-public signals, called “announcement clarity” (Myatt and Wallace, 2014; Arato and Nakamura, 2011). The correlation between two fragmented signals is

\(^7\) Arato and Nakamura (2011) referred to the standard loss function of Morris and Shin (2002). Their conclusions are still dependent on the choice of the payoff structure.
\[ \rho_z = \begin{cases} \frac{\delta}{\gamma + \delta} = \frac{1}{\gamma} & \text{if } i = k \\ 0 & \text{otherwise} \end{cases} \]

Similarly, we compute the correlation coefficient between two private signals
\[ \rho_x = \frac{\beta}{\beta + u} = \frac{1}{\beta} \phi_x \]

We establish an equilibrium strategy and social information allocation matching the aforementioned structure
\[ a^i = \frac{\phi_x(1-r\rho_x)z^i + \phi_x(1-r\frac{\phi_x}{n})x^i}{\phi_x(1-r\rho_x) + \phi_x(1-r\frac{\phi_x}{n})} \]  

(18)

The weight attributed to the semi-public signal is derived from Equation (17)
\[ \lambda_{eq, aa} = \frac{\phi_x(1-r\rho_x)}{\phi_x(1-r\rho_x) + \phi_x(1-r\frac{\phi_x}{n})} \]  

(19)

For \( r > 0 \), agents react to the relatively correlated signals more strongly than the weight given by the first-order expectation \( \frac{\phi_x}{\phi_x + \phi_x} \). Proof in Appendix E demonstrates that the expected loss function under fragmented and ambiguous announcements (aa, henceforth) is expressed as
\[ E(L^M_{aa}) = \frac{1}{\beta u + \beta (1-r) + \gamma \delta} \]  

(20)

If \( \delta \to \infty \) and \( u \to \infty \), Equation (19) boils down to Equation (B.1.3). Higher values of \( \delta \) increase the agents' abilities to predict the induced beliefs of other agents, while an increase in the fragmentation measure \( (n) \) results in an adverse effect. Thus, both instruments are equivalent. A fragmented policy is optimal if and only if
\[ \frac{1}{\rho_z|\delta_1} - \frac{1}{\rho_z|\delta_2} \geq r(1 - 1/n) \]

with
\[ \rho_z|\delta_m = \frac{\delta_m}{\gamma + \delta_m} \quad m = 1,2 \quad \text{such that } \delta_1 < \delta_2 \]

Proof.

Let
\[ E(L^M_{aa} | n = 1, \delta_1)_{aa} = \frac{1}{\beta u + \beta (1-r) + \gamma \delta_1} \]  

and
\[ E(L^M_{aa} | n \geq 2, \delta_2)_{aa} = \frac{1}{\beta u + \beta (1-r) + \gamma \delta_2 (1-\frac{\gamma}{n})} \]

We ascertain that \( E(L^M_{aa} | n = 1, \delta_1)_{aa} \geq E(L^M_{aa} | n \geq 2, \delta_2)_{aa} \) Q.E.D

More ambiguous information lowers its correlation and precision when the clarity is low \((\delta \downarrow)\) and \( Z^i \) is more correlated than \( x^i \). A more ambiguous announcement shifts its correlation to another signal’s correlation. Similarly, when the fragmentation measure is high \((n \uparrow)\), agents rely more on their private information. Both ambiguity and fragmentation contribute equally to enhancing the efficient utilization of information. It is noteworthy that ambiguity leads to a decrease in the accuracy and shared understanding of the information, further reducing information commonality. It would be interesting to break down the information structure into two key components:
accuracy and commonality if we assume endogenous fragmented information. Accuracy refers to the precision of agents’ forecasts regarding the state of fundamentals, while commonality pertains to the correlation of disagreements across agents. One can argue that ambiguous announcements reduce the accuracy and shared understanding of information, while fragmented information provision significantly diminishes the commonality of information. This direction is intriguing not only because of the inherent importance of endogenous information but also because efficacy in information utilization and information collection are interconnected (Angeletos and Pavan, 2007).

5. Experimental evidence

Experimental economics is an evolving tool for its relevance to deriving useful policy recommendations within many contexts. We design a repeated game using a controlled laboratory experiment where the experimenter plays the role of the central bank and the players represent the private agents. Neutrality of the context is however an essential condition to run experiments. A total of 48 subjects participated in the experiment at the Groupe d’Analyse et de Théorie Economique Laboratory. The participants are randomly matched in groups of N=8 players within a computerized session. Treatments of the game vary according to the fragmentation measure (n). The first treatment pertains to the full transparency strategy, while the remaining treatments stand for partial transparency portrayed by a fragmented (semi-public) signal in addition to the private one. The repeated games involve selecting a value for an unknown fundamental state \( \theta \) which is randomly drawn from a close but large interval. The state \( \theta \) stands for inflation, output, interest rate, etc. (semi-)public and private information are independently and identically distributed around \( \theta \). To offset the complexity of subjects’ choices, we use the uniform instead of the normal distribution. The payoff function assumes negative externalities (see sub-section 4.1.3)

\[
U_{j,t} = U(a_{j,t}, \theta_t, \bar{a}_{-j,t}) = \begin{cases} 400 + 10 \times \left[-0.25(a_{j,t} - \theta_t)^2 - 0.75 (a_{j,t} - \bar{a}_{-j,t})^2 \right] & \text{if } U_{j,t} > 0 \\ 0 & \text{if } U_{j,t} < 0 \end{cases}
\]

where \( a_{j,t}, \theta_t, \) and \( \bar{a}_{-j,t} \) stand for the individual action, the true state of fundamentals, and the average action of the opponents at time t. To aid the presentation of the payment function, we multiplied the utility function by 10 and added a fixed monetary amount. Theoretical predictions hold even with these adjustments. Bayesian inference is derived with respect to the mathematical derivations provided in Equations (1)-(6). We postulate the following hypotheses:

Hypothesis 1. The (semi-)public signal is a focal point for fundamentals.

Hypothesis 2. Agents overreact to the (semi-)public signal.

Hypothesis 3. With respect to the payoff function described, a more precise and fragmented measure increases aggregate welfare.

Panel regressions are conducted to fit our purposes. Particularly, we calculate the individual forecast error based on Boonlert et al. (2018):

\[ \text{Forecast}_{j,t} = a_{j,t} - \theta \]

To evaluate whether the semi-public signal acts as a coordinating device (Hypothesis 2), we construct a measure for the disagreement across agents (see Seelajaroen et al., 2019; Andreicovici et al., 2020)

\[ \text{Disagreement}_{j,t} = \frac{1}{N-1} \sum (a_{j,t} - \bar{a})^2 \]

\[ \text{Disagreement}_{j,t} \leq \frac{1}{N-1} \sum (a_{j,t} - \theta)^2 \]

Please see Trabelsi and Hichri (2021) on how the assumption of a uniform distribution does not alter the equilibrium set.
We estimate the following panel regression for either of the above measures, taken as the dependent variable \( y \)

\[
y_{jt} = \alpha + \omega \text{weight}_{jt} + \phi^X_{jt} + \epsilon_{jt}
\]

where ‘weight’ stands for how much an agent \( j \) puts on the (semi-public) signal \( \frac{\sigma_{jt}^2 - \sigma_{jt}^1}{\sigma_{jt}^1} \). It is calculated and derived through Equation (3). \( X \) is a set of controls. We include the round at which the action is taken as well as the average payoff for agent \( j \) and the average action of opponents lagged by one period. To account for possible issues of serial correlation, heteroscedasticity, and cross-section dependence, we use Driscoll and Kraay’s estimator (1998). The results are shown in Table 3. Agents do not seem to account for the (semi-)public signal when anticipating the state of fundamentals (refer to Panel A). The coefficient associated with the variable ‘weight’ is not statistically significant and has the wrong sign. We do not have evidence of the focal role of the (semi-)public signal (Hypothesis 1 is rejected). However, central bank communication has a strong predicting power in shaping participants’ expectations of the consensus forecast. Particularly, when public information is fully available, the coefficient of the variable "weight" is negative as expected and statically significant at 1%, meaning that participants overreact to public information (Hypothesis 2 not rejected) but they tend to invest less on the same information as the fragmentation measure \( (n) \) increases. The observations are partially consistent with the theoretical predictions. It is noteworthy that participants’ behavior is dictated by incentives and learning mechanisms. Learning through others’ actions reduces the individual forecast error but does not affect the disagreement measure while switching from round to round has a significant effect when public information is fully available. Monetary incentives portrayed by the variable "Own average payoff_lag" have a significant and negative effect on the dispersion between participants’ opinions. Participants take advantage of past information to be close to the average opinion. As public information is fragmented, participants’ behavior is a bit disturbed, reflecting their diverse and limited cognitive capacities. This is consistent with k-level reasoning theory (see Nagel, 1995; Stahl and Wilson, 1994; Stahl and Wilson, 1995, etc.).

<table>
<thead>
<tr>
<th>Variables</th>
<th>Panel A: Individual forecast error</th>
<th>Panel B: Disagreement to the consensus forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Treatment 1</td>
<td>Treatment 2</td>
</tr>
<tr>
<td></td>
<td>( n=1 )</td>
<td>( n=2 )</td>
</tr>
<tr>
<td>weight</td>
<td>-0.7941</td>
<td>0.0488</td>
</tr>
<tr>
<td></td>
<td>(-0.589)</td>
<td>(0.044)</td>
</tr>
<tr>
<td>Trial</td>
<td>0.1965**</td>
<td>0.0142</td>
</tr>
<tr>
<td></td>
<td>(2.616)</td>
<td>(0.181)</td>
</tr>
<tr>
<td>Opponents’ action_lag</td>
<td>-0.0016***</td>
<td>-0.0003</td>
</tr>
<tr>
<td></td>
<td>(-3.098)</td>
<td>(-0.695)</td>
</tr>
<tr>
<td>Own average payoff_lag</td>
<td>-0.0037</td>
<td>-0.0294*</td>
</tr>
<tr>
<td></td>
<td>(-1.631)</td>
<td>(-1.863)</td>
</tr>
<tr>
<td>_cons</td>
<td>0.7107</td>
<td>8.1103</td>
</tr>
<tr>
<td></td>
<td>(0.480)</td>
<td>(1.735)</td>
</tr>
<tr>
<td>N° observations</td>
<td>657</td>
<td>705</td>
</tr>
</tbody>
</table>

Notes: \( t \) statistics in parentheses. * \( p < 0.1 \), ** \( p < 0.05 \), *** \( p < 0.01 \)

---

9We obtain the same results qualitatively when we use the (semi-)public signal instead of the “weight” variable. Results are available upon request.
For Hypothesis 3, we calculate the welfare as the average of rewards over all agents by a group of matching individuals at time t. Further, we measure central bank transparency through two components (the fragmentation measure $n$ and the precision of the (semi-)public signal. We assess the precision of information through two metrics. First, we follow Middeldorp et al. (2011) and related experiments and take the inverse of the variance of the drawn semi-public signals observed by each subject $j$. To aid the presentation of the result, we multiply the resulting measure by 10000. The second metric is a standard measure of accuracy which equals the absolute difference between the observed (semi-)public signal by agent $j$ to the true state of economy $\theta$.

We, then, regress the social welfare on central bank transparency (CBT) using the following regression:

$$Welfare_{j,t} = \delta + \pi CBT_{j,t} + \mu_{j,t}$$

We use the Generalized Least Square (GLS) estimator since it accommodates large T panels and time-invariant regressor(s). We display the associated results in Table 4. The social welfare is muted to both transparency components, leading to the rejection of Hypothesis 3. An increase in the fragmentation measure ($n$) by 1% engenders a decrease in the overall welfare by 17.5 to 18.3%. More precise (semi-)public information reduces welfare by 166% for the first metric and by 9.1% for the second metric. Yet, the results testify to the absence of a trade-off and show that partial transparency is suboptimal. However, the central bank should make less precise public information. They results signal the importance of weighing central bank transparency based on quantity and quality whenever a central bank decides about the optimal levels.

### Table 4. Impact of central bank transparency on social welfare.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Welfare</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fragmentation measure: n</td>
<td>-18.3028***</td>
<td>-17.5568***</td>
</tr>
<tr>
<td></td>
<td>(-13.744)</td>
<td>(-14.188)</td>
</tr>
<tr>
<td>Precision of (semi-)public signal (1)</td>
<td>-166.0471**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.206)</td>
<td></td>
</tr>
<tr>
<td>Precision of (semi-)public signal (2)</td>
<td></td>
<td>-9.1326***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-21.994)</td>
</tr>
<tr>
<td>_cons</td>
<td>308.5210***</td>
<td>329.0690***</td>
</tr>
<tr>
<td></td>
<td>(23.937)</td>
<td>(83.640)</td>
</tr>
<tr>
<td>N° observations</td>
<td>2160</td>
<td>2160</td>
</tr>
</tbody>
</table>

Notes: $t$ statistics in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

### 6. Discussion and conclusion

Central banks have endeavored to increase the transparency of their decision-making processes. Yet, transparency is no panacea and may have severe consequences.

Public information is an anchoring point for fundamentals but outperforms private information with its coordinating ability. Nevertheless, disclosing all available information is not always optimal (Moreno and Takalo, 2016; Salle et al., 2019). If financial market participants overestimate the central bank's views and do not consider what they reflect as noisy signals, communication may be detrimental. We show that a fragmented policy reduces the overreaction to public information but improves social welfare only under certain conditions. Indeed, there are instances where the central bank prefers rather less accurate communication (see Table 5). Ambiguity and fragmentation are both key factors in improving the efficient use of information. However, ambiguity often leads to decreased accuracy and shared understanding, ultimately reducing information commonality. When examining endogenous fragmented information, it becomes particularly interesting to dissect the information structure into
accuracy and commonality. Here, accuracy relates to how precisely agents forecast the state of fundamentals, while commonality involves the correlation of disagreements among agents. While experimental data favor a partial transparency strategy, there is a strict move towards disclosing full and less accurate public information to the private sector. Overall, the results suggest that the optimal strategy should balance the benefits and the costs of transparency in terms of quantity and quality. How agents perceive the content of the message might also explain heterogeneous behavior. Much of our experimental observations are driven by the presence of less sophisticated players. Moreover, it is of ultimate importance to explore the implications of central bank transparency on economic agents, particularly those in a learning environment (Berardi and Duffy, 2007). Trabelsi and Hichri (2021), for example, demonstrated that convergence to equilibrium in the fragmented information game is driven by reinforcement and belief-based learning.\(^\text{10}\)

Policy implications from the findings suggest that central banks should adopt a subtle transparency approach, recognizing its limitations, and ensuring communication strategies are adaptable to changing economic conditions (Muchlinski, 2022). Whenever there are threats to financial stability such as the crisis times, it would be impartial to opt for partial transparency provided by a fragmented information policy. Central bank practices should incorporate agents’ perceptions and the degree of trustfulness about the different communication tools’ contents (Bholat et al., 2019). During crises, partial transparency may be warranted to address threats to financial systems, while full transparency is often the default in normal times (Fratzscher et al., 2010). This is especially pertinent when communicating about macroprudential policies. The choice of transparency strategy also relates to the level of economic development. For instance, compared to the US Federal Reserve and the European Central Bank, there is a greater need for increased transparency in emerging and developing countries. Therefore, partial transparency strategies may be more suitable for countries where central bank transparency levels are already high (see Dincer et al., 2022).

Furthermore, central bank credibility and reputation are warranted when undertaking any form of transparency strategy. Assuming fragmented information in financial markets implies the likelihood of a social interaction transmission between agents in financial markets (see Hong et al., 2005; Brown et al., 2008; and Argan et al., 2014). Effective communication can amplify the impact of a rumor on mobilization, sometimes even surpassing the impact of a fully believed story (Chen et al., 2016).

A research avenue within the class of abstract games as considered in this paper is to introduce frictions, that is a situation where agents are incapable of reaching the average dispersion about the unknown state of the economy (see Angeletos and Lian, 2016).

The trade-off or the conflicting welfare effect of information may re-appear in a more extended setting. One step in this direction might be examining it under a structure where both semi-public and private information are endogenous (see Tamura, 2015, Tamura, 2018). Ambiguity and fragmentation are both key factors in improving the efficient use of information. However, ambiguity often leads to decreased accuracy and shared understanding, ultimately reducing information commonality. When examining endogenous fragmented information, it becomes particularly interesting to dissect the information structure into accuracy and commonality. Here, accuracy relates to how precisely agents forecast the state of fundamentals, while commonality involves the correlation of disagreements among agents. So far, our framework provides valuable outcomes but they are derived from an abstract game. To overcome this limitation, we need to spell out concrete applications as everything should be formalized from micro-foundations (Angeletos et al., 2016). Our theory is interpreted in the context of monetary

\(^{10}\) Reinforcement learning describes well the case of an agent who learns through repetition, called experiential learning. Belief-based learning fits an agent who observes what others do and makes thoughts about their future actions (see Feltovich, 2003).
policy and industrial organization, but it can also be extended to model communications in bank runs, financial crises, and political revolution. We hope that our current contribution has scrutiny of welfare information effects though.

Further theoretical scenarios involving fragmented information discussed in this paper are left for experimental investigation on the grounds of funds’ availability. Efforts in this direction are evolving though.\textsuperscript{11} Until this happens, questioning central bank transparency never gets old.

### Table 5. States beyond which fragmented information with high precision is preferred.

<table>
<thead>
<tr>
<th>Information structure</th>
<th>Loss function type</th>
<th>Fragmentation policy is preferred if...</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n ) semi-public signal + 1 private signal</td>
<td>Loss function with no direct externalities</td>
<td>( \frac{\gamma_2}{\gamma_1} \geq \frac{1 - \frac{r}{n}}{1 - r} )</td>
</tr>
<tr>
<td>( n ) semi-public signals + 1 private signal</td>
<td>Loss function with positive externalities</td>
<td>( \frac{\gamma_2}{\gamma_1} \geq \frac{1 + r - 2 \frac{r}{n}}{1 - r} )</td>
</tr>
<tr>
<td>( n ) semi-public signals + 1 private signal</td>
<td>Loss function with negative externalities</td>
<td>No condition. More precise fragmented information is optimal.</td>
</tr>
</tbody>
</table>

**Note:** The third column indicates the thresholds above which fragmented information with high precision is the preferred strategy. \( \gamma_1 \): low precision. \( \gamma_2 \): high precision. \( \rho_{Z|\delta_m} \) is the correlation between two semi-public signals.

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**Conflict of interest**

All the authors claim that the manuscript is completely original. The authors also declare no conflict of interest.

**Author contributions**

Conceptualization: Emna Trabelsi; Investigation: Emna Trabelsi; Methodology: Emna Trabelsi; Formal analysis:
Appendix

A. Derivation of Equation (5).

The best linear response of an agent $j$ is

$$E_j^i(\theta) = \frac{\gamma Z_i + \beta x_j^i}{\gamma + \beta} , \text{ with } i=1, 2, ..., n$$

(A.1)

And average expected action

$$E_j^i(\bar{a}) = E_j^i(\lambda Z + (1-\lambda)\theta) = \lambda \left(\frac{n-1}{n} E_j^i(\theta) + \frac{1}{n} Z_i \right) + (1-\lambda)E_j^i(\theta) = \lambda \left(\frac{n-1}{n} \frac{\gamma Z_i + \beta x_j^i}{\gamma + \beta} + \frac{1}{n} Z_i \right) + (1-\lambda) \frac{\gamma Z_i + \beta x_j^i}{\gamma + \beta} =$$

$$\left(\frac{\gamma}{\gamma + \beta} + \frac{\lambda \beta}{n (\gamma + \beta)}\right) Z_i + \left(1 - \left(\frac{\gamma}{\gamma + \beta} + \frac{\lambda \beta}{n (\gamma + \beta)}\right)\right) x_j^i$$

(A.2)

Plugging Equation (A.1) and Equation (A.2) into Equation (1) yields

$$a_j^i = (1 - r) \frac{\gamma Z_i + \beta x_j^i}{\gamma + \beta} + r \left[ \left(\frac{\gamma}{\gamma + \beta} + \frac{\lambda \beta}{n (\gamma + \beta)}\right) Z_i + \left(1 - \left(\frac{\gamma}{\gamma + \beta} + \frac{\lambda \beta}{n (\gamma + \beta)}\right)\right) x_j^i \right]$$

(A.3)

Rearranging terms leads to Equation (5).

B.1 Derivation of the loss functions and the weight attached to the semi-public information in Equation (6).

Case 1: AR: “Actions get right”

$$E(L_{AR}^P) = E_j^i \left[(a_j^i - \theta)^2 \right] = \frac{\lambda^2}{\gamma} + \frac{(1-\lambda)^2}{\beta} = \frac{\gamma + \beta(1-\gamma)^2}{\gamma + \beta(1-\gamma)^2}$$

(B.1.1)

Case 2: RH: “Reducing heterogeneity”

$$E(L_{RH}^P) = E_j^i \left[E_h^k(\alpha_h^k - a_j^i)^2 \right] = 2 \left[\frac{\lambda^2}{\gamma} \left(1 - \frac{1}{n}\right) + \frac{(1-\lambda)^2}{\beta} \right] = \frac{2\left[\gamma \left(1 - \frac{1}{n}\right) + \beta(1-\gamma)^2\right]}{\gamma + \beta(1-\gamma)^2}$$

(B.1.2)

Case 3: M: The mixture consists of the weighted sum of losses in case 1 and case 2
An alternative way to derive Equation (6) comes through the loss function in Equation (B.1.3)

Agents minimize their loss function

$$E(L^p_M) = (1 - r)E_j^i\left[(a^i_j - \theta)^2\right] + \frac{r}{2} E_j^i\left[E_h^k(a_h^k - a_j^i)^2\right]$$

s.t. \(a^i_j = \lambda Z_i + (1 - \lambda)x^i_j\)

$$E(L^p_M) = (1 - r)\left[\frac{\lambda^2}{\sigma} + \frac{(1-\lambda)^2}{\beta}\right] + \frac{r}{2} 2\left[\frac{\lambda^2}{\sigma} + (1 - \frac{1}{n}) + \frac{(1-\lambda)^2}{\beta}\right] \quad (B.1.4)$$

Differentiating Equation (B.1.4) with respect to \(\lambda\) implies

$$\frac{\partial E(L^p_M)}{\partial \lambda} = 2\frac{\lambda}{\sigma}(1 - \frac{r}{n}) - 2\frac{(1-\lambda)}{\beta} = 0 \iff \lambda_{eq} = \frac{\sigma}{\sigma + \beta(1-\frac{r}{n})}$$

The second-order condition pertains to

$$\frac{\partial^2 E(L^p_M)}{\partial \lambda^2} = 2\frac{1}{\sigma}(1 - \frac{r}{n}) + 2\frac{1}{\beta} > 0$$

**B.2** Derivation of the loss function and the weight attached to semi-public information in the case of positive externalities.

$$E(L^p_M)_{s} = (1 - r)E_j^i\left[(a^i_j - \theta)^2\right] + r E_j^i\left[E_h^k(a_h^k - a_j^i)^2\right] \quad (B.2.1)$$

where

$$\begin{align*}
E_j^i\left[(a^i_j - \theta)^2\right] &= \frac{\lambda^2}{\sigma} + \frac{(1-\lambda)^2}{\beta} = \frac{\gamma(1+r)^2 + \beta(1+r-2\gamma)^2}{\gamma(1+r) + \beta(1+r-2\gamma)^2} \\
E_j^i\left[E_h^k(a_h^k - a_j^i)^2\right] &= 2\left[\frac{\lambda^2}{\sigma}(1 - \frac{1}{n}) + \frac{(1-\lambda)^2}{\beta}\right] = 2\frac{\gamma(1+r)^2(1-\gamma) + \beta(1+r-2\gamma)^2}{\gamma(1+r) + \beta(1+r-2\gamma)^2} \quad (B.2.2)
\end{align*}$$

Using Equation (B.2.1) and Equation (B.2.2)
\[
E(L_{pe}^{PS}) = (1 - r)E_j^i[(a_j^i - \theta)^2] + rE_j^i \left[ E_h^k (a_h^k - a_j^i)^2 \right] = \frac{(1+r)(1+r-\frac{r}{n})}{\gamma(1+r) + \beta(1+r-\frac{r}{n})}
\] (B.2.3)

The loss function with positive externalities is a decreasing function of \( \gamma \) and \( \beta \) and an increasing function of \( n \).

We solve for Equation (11) as follows

\[
E(L_{pe}^{PS}) = (1 - r)E_j^i[(a_j^i - \theta)^2] + rE_j^i \left[ E_h^k (a_h^k - a_j^i)^2 \right] = (1 - r) \left[ \frac{\lambda^2}{\gamma} + \frac{(1 - \lambda)^2}{\beta} \right] + 2r \left[ \frac{1}{\gamma} \left( 1 - \frac{1}{n} \right) + \frac{(1 - \lambda)^2}{\beta} \right]
\]

Partial derivative with respect to \( \lambda \)

\[
\frac{\partial E(L_{pe}^{PS})}{\partial \lambda} = 2 \frac{1}{\gamma} \left( 1 + r - 2 \frac{r}{n} \right) - 2 \frac{(1 - \lambda)}{\beta} (1 + r) = 0 \Leftrightarrow \lambda_{eq,pe} = \frac{\gamma(1 + r)}{\gamma(1 + r) + \beta \left( 1 + r - 2 \frac{r}{n} \right)}
\]

The second-order condition suggests

\[
\frac{\partial^2 E(L_{pe}^{PS})}{\partial \lambda^2} = \frac{2}{\gamma} \left( 1 + r - 2 \frac{r}{n} \right) + 2(1 + r) \frac{1}{\beta} > 0 \quad Q.E.D
\]

C. Proof of propositions 1 & 2 & 3.

We depart from the loss function of Equation (B.1.3) and denote

\( \{ y_1: \text{low precision} \} \quad \{ y_2: \text{high precision} \} \)

The expected loss function under full publicity (\( n=1 \)) and low precision (\( y_1 \)) is expressed as

\[
E(L_{M/n = 1, y_1}^{PS}) = \frac{1}{\frac{1}{1-r} + \beta}
\] (C.1.1)

Expected loss under fragmented information (\( n \geq 2 \)) with high precision (\( y_2 \))

\[
E(L_{M/n \geq 2, y_1}^{PS}) = \frac{1}{\frac{1}{1-r} + \beta}
\] (C.1.2)

\[
E(L_{M/n = 1, y_1}^{PS}) \geq E(L_{M/n \geq 2, y_2}^{PS}) \quad \Leftrightarrow \quad \frac{y_2}{y_1} \geq \frac{1 - r/n}{1 - r} \quad Q.E.D
\]

The loss function is given by Equation (B.2.2)

The expected loss function under full publicity (\( n=1 \)) and low precision (\( y_1 \)) is written as
\[ E(L_M^n/n = 1, \gamma_1)_{pe} = \frac{(1+r)(1-r)}{\gamma_1(1+r)+\beta(1-r)} \]  

(C.2.3)

Expected loss under fragmented information \((n \geq 2)\) with high precision \((\gamma_2)\) is as follows

\[ E(L_M^n/n \geq 2, \gamma_2)_{pe} = \frac{(1+r)(1+r-2\gamma_2)}{\gamma_2(1+r)+\beta(1+r-2\gamma_2/n)} \]  

(C.2.4)

\[ E(L_M^n/n = 1, \gamma_1)_{pe} \geq E(L_M^n/n \geq 2, \gamma_2)_{pe} \iff \frac{\gamma_2}{\gamma_1} \geq \frac{1+r-2r/n}{1-r} \]

Q.E.D

Under negative externalities and by introducing Equations (1)-(6), expected loss function is pinned down to

\[ E(L_M^n)_{ne} = \frac{1}{\gamma + \beta(1 - \gamma/n)} \]

If \(\gamma \uparrow\) and \(n \uparrow\), \(E(L_M^n)_{ne} \downarrow\) Q.E.D

D. Introducing nonlinear costs.

We assume that costs are positive and unbounded in line with Demertzis and Hoeberichts (2007).

\[ C^{PS}(\beta) = c\beta^{\kappa}, \quad c > 0 \text{ and } \kappa > 1 \]

The expected loss function is given by

\[ E(T_M^n) = E(L_M^n) + C^{PS}(\beta) = \frac{1}{\gamma + \beta(1 - \gamma/n)} + c\beta^{\kappa} \]  

(D.1)

The first-order condition of Equation (D.1) is

\[ \frac{\partial E(T_M^n)}{\partial \beta} = -\frac{1}{(\gamma + \beta)^2} + \kappa c\beta^{\kappa-1} = 0 \]  

(D.2)

Then, the second-order derivative establishes \(\frac{\partial^2 E(T_M^n)}{\partial \beta^2} > 0 \iff \frac{2(1-'\gamma')^2}{[\gamma + \beta(1-'\gamma'/n)]} + \kappa(\kappa-1)c\beta^{\kappa-2} > 0\) which is always satisfied.

The implicit theorem function provides the following
Introducing non-linear costs confirms the strategic substitutability of the public and private signals. The private signal is an increasing function of the fragmentation measure \((n)\). 

E. Equilibrium under imperfect correlated signals. 

Recall 

\[ a_j^f = (1 - r)E_j^f(\theta) + rE_j^f(\bar{a}) \]  
(E.1) 

and 

\[ a_j^f = \lambda Z_j^f + (1 - \lambda)x_j^f \]  
(E.2) 

with 

\[ E_j^f(\bar{a}) = \lambda E_j^f(Z) + (1 - \lambda)E_j^f(x) \] 

where 

\[ 
\begin{align*} 
E_j^f(\theta) &= \frac{\varphi_x Z_j^f + \varphi_x x_j^f}{\varphi_x + \varphi_x} \\
E_j^f(Z) &= \left(1 - \frac{\rho_z}{n}\right) E_j^f(\theta) + \frac{\rho_z}{n} Z_j^f \\
E_j^f(x) &= \frac{\varphi_x (1 - \rho_x Z_j^f + \varphi_x (1 - \rho_x x_j^f)}{\varphi_x + \varphi_x} 
\end{align*} 
\]  
(E.3) 

We explicit \(E_j^f(\theta)\) and \(E_j^f(\bar{a})\) as they figure in Equation (E.3) and equalize coefficients in Equation (E.1) and Equation (E.2). Thereby, 

\[ \lambda_{eq,aa} = \frac{\varphi_x (1 - r\rho_x)}{\varphi_x (1 - r\rho_x) + \varphi_x (1 - r\rho_x)} \]  
(E.A) 

\[ \begin{align*} 
\varphi_x &= \frac{\gamma \delta}{\gamma + \delta} \\
\varphi_x &= \frac{\beta u}{\beta + u} 
\end{align*} \] 

We introduce Equation (E.A) in Equation (B.1.4) to obtain Equation (22).
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