Journal of Economic Analysis 2025 4 (3) 90-110

Journal of Economic Analysis

Homepage: https://www.anserpress.org/journal/jea

Convexity of the Phillips Curve and Difficulty of Monetary Policy to Fight against Inflation

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ABSTRACT

We propose a theoretical model that underlines the implications of a non-linear Phillips curve for the difficulty of monetary policy to fight against inflationary tensions similar to those encountered in 2021 and 2022 at a worldwide level, due to the resumption of demand after the COVID crisis and to the war in Ukraine. In such non-linearities, the interest rate becomes an asymmetric and convex function of an inflationary supply shock variation. In the case of inflationary tensions, the nominal interest rate can then strongly increase above its long-term equilibrium value. Economic recession is exacerbated, whereas inflationary tensions are more accentuated. Then, the risk is a more than proportional increase in public expenditure to compensate for the decrease in private consumption, which could not be achieved without a strong growth in the public indebtedness level.

KEYWORDS

Inflation; Phillips Curve; Convexity; Non-linearity; Monetary Policy

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ISSN 2811-0943
doi: 10.58567/jea04030005

Received 22 February 2025; Accepted 23 May 2025; Available online 9 June 2025; Version of Record 15 September 2025

1. Introduction

The recent period witnessed an unexpected and 'missed inflation' phenomenon: indeed, in 2021 and 2022, inflation rates reached levels unknown since the 1970s, despite the recession due to the COVID-19 crisis and afterward to the war in Ukraine. These high inflation rates followed a long period of 25 years of 'missed disinflation', where inflation rates have strangely remained low during the 'Great Moderation', despite the increase of the output-gap (see Figures 1 and 2).

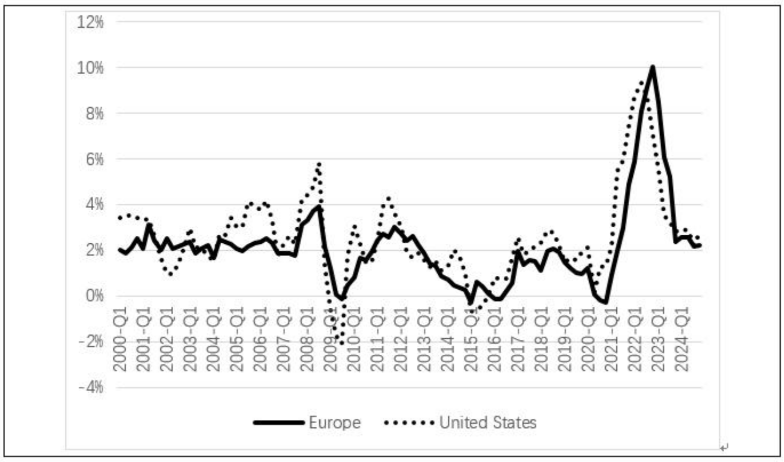

Figure 1. Inflation rates in the European Union and the United States.

Source: United States: Consumer Price Index, all Items, wage earners, FRED Database. Europe: Harmonized Index of Consumer Prices, EUR19, EUROSTAT data

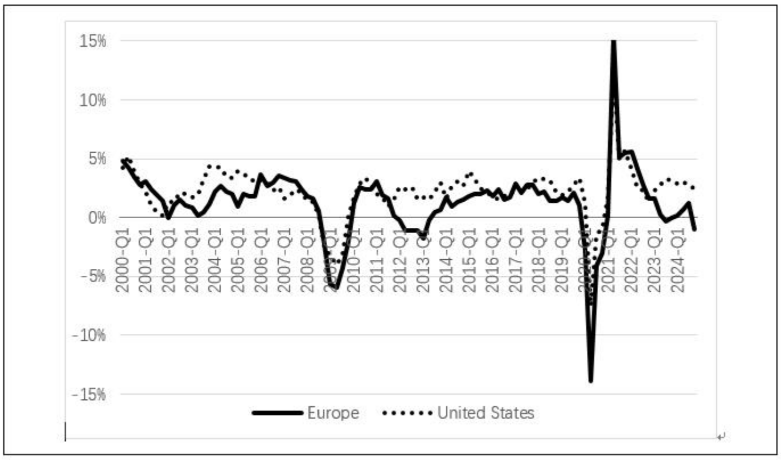

Figure 2. Nominal GDP Growth Rates in the European Union and in the United States.

Source: United States: Real Gross Domestic Product, Billions of Chained 2017 Dollars, FRED Database. Europe: GDP at constant prices, chain-linked volumes, EUR19, EUROSTAT data.

Therefore, the link between the inflation rate and GDP growth appears quite complex and difficult to understand. According to the statistical findings of Phillips in 1958, there would be a correlation between reduced unemployment and increasing wages in an economy, at least in the short run. However, since the 2000s, the slope of the Phillips curve seems to have declined, and there has been controversy over the usefulness of this Phillips curve in predicting inflation. Besides, the hypothesis of non-linearities in the Phillips curve has recently renewed interest in the economic literature.

For example, for the United States, Jorgensen and Lansing (2024) assume that the 'original' Phillips curve, defined as the statistical relationship between the level of inflation and economic activity, has steepened from 2007-Q4 to 2019-Q2. However, the economic literature, as highlighted in a survey by Furlanetto and Lepetit (2024), typically finds that over the past few decades, the reduced-form 'accelerationist' Phillips curve, which relates changes in inflation to economic activity, has flattened. The authors clarify this apparent contradiction by considering variations in the degree of anchoring of agents' inflation forecasts. Specifically, imperfectly anchored inflation expectations, combined with an inflation-targeting central bank, create an upward bias in the slope of the accelerationist Phillips curve and a downward bias in the slope of the original Phillips curve relative to the true structural slope of the New Keynesian Phillips Curve. Improved anchoring mitigates both biases, suggesting that better anchored expectations could account for the flattening of the accelerationist Phillips curve alongside reductions in inflation volatility and persistence.

The importance of expectations is also underlined by Doser *et al.* (2023). They examine the presence of nonlinearities in the Phillips curve. In the United States, over the estimation period between 1968 and 2019, they show that the linear model cannot be rejected if we properly control inflation expectations. So, not controlling for consumer expectations may lead the econometrician to overestimate the degree of nonlinearity. On the contrary, if such expectations are taken into account, a piecewise-linear Phillips curve, containing thresholds, added not much to the econometrician's estimation.

Estimations of the Phillips curve have also been made for European countries. For the period 2000Q1-2022Q2, for 24 advanced European economies and 7 European emerging economies, Baba *et al.* (2023) highlight significant differences in Phillips curve parameters across European economies. Inflation seems to be more sensitive to domestic slack and external price pressures in emerging European economies compared to their advanced counterparts, contributing to a greater pass-through of global commodity price shocks into domestic prices, and, consequently, to larger increases in inflation rates. Across Europe, inflation also appears to have become increasingly backward-looking and more sensitive to commodity price shocks since the onset of the COVID-19 pandemic. All these characteristics explain the width of the surge in inflation rates in 2022. Between 1970 and 2005, Musso *et al.* (2009) find strong evidence of time variation in the mean and slope of the Phillips curve occurring in the early to mid-1980s in the European Union, but not in inflation persistence once the mean shift is allowed. As a result of the structural change, the Phillips curve became flatter around a lower mean of inflation.

Furthermore, using 11 Eurozone countries between 1999 and 2017, Ho and Njindan Iyke (2019) find threshold effects for the Phillips curve. They find that the relationship between inflation and unemployment is only negative when unemployment is lower than 5%. The negative relationship turns positive when unemployment is between 5% and 6.54%. Finally, inflation and unemployment are unrelated once a threshold of 6.54% unemployment rate is surpassed. In the same way, using sectoral data from 24 advanced economies in Europe, between 2012 and 2019, Ari *et al.* (2023) show that after the COVID-19 pandemic, higher digitalization (weight of the e-commerce and stronger price flexibility) and lower trade intensity were associated with steeper Phillips curves. So, post-pandemic Phillips curve estimates indicate some steepening in the UK, Spain, Italy, and the Euro area as a whole, but at magnitudes that are too small to explain the entire surge in inflation in 2021–22. On the contrary, the Phillips curve would have flattened in France and Germany.

Therefore, the economic literature has largely studied the reality of the existence of a negative relationship between unemployment and inflation, as mentioned by Phillips in 1958. What is the strength of the empirical validation of such a relationship? And if such a relationship does exist, is it linear, or can we discover non-linearities in the Phillips curve? In the case of non-linearities, this paper shows that the interest rate becomes an asymmetric and convex function of the variation in an inflationary supply shock. Indeed, in the case of inflationary tensions, the nominal interest rate can strongly increase above its long-term equilibrium value. Economic recession is exacerbated, whereas inflationary tensions are more accentuated. Then, the risk is a more than proportional increase in public expenditure to compensate for the decrease in private consumption, which could not be achieved without a strong growth in the public indebtedness level.

The rest of the paper is as follows. The second section recalls the economic literature about the potential non-linearity in the Phillips curve. The third section describes the basic equations of our model: a traditional DSGE model, but whose specificity relies on a nonlinear Phillips curve. The fourth section describes the results of the model in the case of a non-linear and convex Phillips curve. The fifth section concludes the paper.

2. Economic literature on a non-linear Phillips curve

Since the 1990s, the economic literature has emphasized the possibility of a non-linear Phillips curve by allowing for convexities in the transmission mechanism between the output gap and inflation. More specifically, according to this literature, positive deviations of aggregate demand from potential (the case of an upswing or 'boom') would be more inflationary than negative deviations (recessions) are disinflationary. This would conform with the theoretical framework of the Keynesian literature: under conditions of full employment, inflation responds strongly to demand conditions, whereas in deep recessions, it is relatively insensitive to changes in activity.

Theoretically, Speigner (2014) finds that in a linear Phillips curve model, it is only the short-term unemployment rate that matters for wage dynamics. However, once convexities are allowed during the estimation process, long-term unemployment has a significant negative effect on wage inflation. Indeed, long-term unemployment rises more slowly than short-term unemployment during a recession, and weaker wages are negotiated than those required by the true economic situation. So, if the true model were convex, there could remain significant downward wage pressure from long-term unemployment even after the short-term unemployment gap has closed. When long-term unemployment is high, downward resistance to wage reduction implies a flattening of the Phillips curve, and unemployment is then quite inefficient in reducing wage inflation further. Therefore, many empirical studies have tried to test the hypothesis of a non-linear Phillips curve.

For example, for the United States, Clark *et al.* (1996) assess a significant asymmetry in the U.S. output-inflation process between 1964 and 1990. Estimating a Phillips curve where a positive output gap can have much more inflationary consequences, they show that excess demand conditions are much more inflationary than excess supply conditions are disinflationary. The important policy implication of this asymmetry is that an overheating of the economy can be very costly, because it will necessitate a severe tightening in monetary conditions to re-establish inflation control. Policy rules that fail to guard against overheating will result in significantly larger monetary business cycles and permanent losses in output. Besides, in tight periods of inflationary tensions, a contractionary monetary policy would be more effective in reducing inflation than in a linear model.

In the same way, Barnes and Olivei (2003) support the hypothesis of a piecewise linear Phillips curve, the curve shifting when the unemployment rate is outside a range of conventional and expected values. Indeed, empirical estimations show that in the United States, between 1961 and 2002, if the unemployment rate is below the threshold value of 4% or above the value of 7.5%, there is a trade-off between inflation and unemployment, and inflation responds more strongly to unemployment. Cristini and Ferri (2021) also find evidence of a convex US Phillips curve between 1961 and 2019. Methods based upon a piecewise approach and strengthened by a threshold technique empirically clearly support the existence of convexities (and not concavities) as well as discontinuities in this curve. According to the authors, a regime-switching macro model would better fit these data; for example, with a bad state of nature and a slack labor market (the unemployment rate is above a threshold), or with a good state of nature and a tight labor market.

Besides, Demirel and Wilson (2023) estimate a non-linear Phillips curve for the United States, and find that the interaction between supply-chain disruptions and low economic slack (low unemployment and tight labor markets) amplified the effects of expansionary fiscal policies on inflation during 2020 and 2021. This interaction may explain the inflationary pressures that arose during the strong rebound from the COVID-19 crisis and recession. Indeed, the authors find that changes in overall demand exert larger effects on inflation in periods of constrained supply. Therefore, the slope of the Phillips curve would have increased in 2020 and 2021 due to both supply disruptions and diminishing slack. Demirel and Wilson (2023) demonstrate that allowing the slope term to vary with supply conditions enhances the out-of-sample predictive accuracy of the Phillips curve across various measures of slack.

For a larger set of countries, for 31 mostly advanced economies over the period 1996-2017, Forbes et al. (2021) also show that when output exceeds potential, the upward pressure on prices (from reductions in slack) is far greater than any equivalent downward pressure (from increases in slack) when output is below potential. Then, the Phillips curve would be non-linear when inflation is low, below 3% ('low inflation bend' curve because of price and wage downward rigidities), but linear and sharp for higher inflation rates, when there is no spare capacity.

Many papers have then studied the implications of such a non-linear Phillips curve for the optimal monetary policy, and its implications for the accentuation of business cycles.

In this context, the linear-quadratic paradigm underlying Taylor rules for the definition of the optimal monetary policy can be challenged. Indeed, Keynesian hypotheses have underlined that even when nominal wages are flexible upwards, they may be rigid downwards, introducing convexity in the inflationary dynamic. This can explain why the Phillips curve is probably nonlinear in Europe, whereas it is more linear in the US, where the labor market is more flexible [see Dolado *et al.* (2005)]. Furthermore, politically, regarding the policymakers' preferences, there may be a greater aversion to recessions than expansions. So, in this context, the optimal monetary policy also becomes non-linear.

For example, Dolado *et al.* (2005) investigate the implications of a nonlinear Phillips curve for deriving optimal monetary policy rules. Combined with a quadratic loss function, they find that the optimal policy is also nonlinear, with the policymaker increasing interest rates by a larger amount when inflation or output is above target than the amount it will reduce them when it is below target. This leads to the inclusion of the interaction between expected inflation and the output gap in an otherwise linear Taylor rule. However, between 1980-89 and 1997-2001, the authors find empirical support for this type of asymmetries in the interest rate-setting behavior of four European central banks (Germany, France, Spain, and a 'virtual' ECB) but none for the US Fed. Nevertheless, the non-linearity in the Phillips curve could explain the huge increase in interest rates in the United States and Europe in 2022 and 2023 (see Figure 3). Therefore, the third and fourth sections try to confirm the theoretical validity of this hypothesis with the help of a simple macro-economic model.

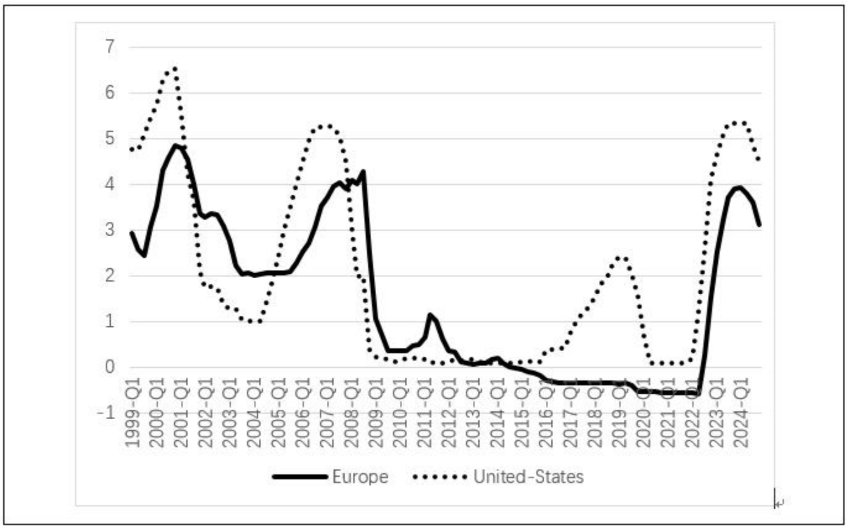

Figure 3. Nominal Interest Rates in the European Union and in the United States.

Note: Interest rates, immediate rates (<24h), call Money/Interbank Rate, %, Quarterly data, not seasonally adjusted. Source: FRED Database.

3. The model

In this section, we propose a theoretical model which underlines the implications of a non-linear Phillips curve for the difficulty of monetary policy to fight against inflationary tensions similar to those encounter in 2021 and 2022 at a worldwide level, due to the resumption of demand after the COVID crisis and to the war in Ukraine. We consider a small standard DSGE model, where budgetary expenditure is defined by the fiscal authority, whereas the short-term nominal interest rate is defined by the monetary authority and is the instrument of the central bank. The specificity of the current paper is to introduce a nonlinear Phillips curve for the determination of prices in this standard framework: inflation is defined by an accelerationist Phillips curve. In the rest of the paper, all lower-case letters with a circumflex accent denote variables in logarithms and variations from their long-run equilibrium values.

3.1. Demand equation and economic policies

Microeconomic-founded equations in traditional New Keynesian models are the following. The intertemporal choice of the representative consumer leads to the following variation in the consumption level:

$$\widehat{c_t} = E_t(\widehat{c_{t+1}}) - [i_t - E_t(\pi_{t+1})] \tag{1}$$

With: (c_t): real consumption of private goods; (i_t): nominal interest rate; (π_t): consumer inflation.

Therefore, the paper posits the hypothesis of rational expectations: individuals make decisions based on all available information to form expectations about future events and make choices. They predict outcomes that do not differ systematically from the market equilibrium, given that they do not make systematic errors when forecasting the future. We acknowledge the existence of limited available information and human error. However, this hypothesis diminishes the role of irrational bubbles in financial markets in the derivation of our model's results.

Besides, on the goods market, the market-clearing condition states that in a free market, prices are sufficiently flexible and can adjust instantaneously to reach the equilibrium between supply and demand of goods and services. In this model, we don't suppose perfect information in the short run, but we hypothesize that markets can approach efficient outcomes as information is discovered. We consider a medium-term temporal horizon, where shortages from the demand or supply side of the market can largely be absorbed. If we also suppose the stability of private investment and net exports, variation of global demand depends on the increase of consumption and public expenditure:

$$\widehat{y_t} = \chi\widehat{c_t} + (1-\chi)\widehat{g_t} \tag{2}$$

With: $(\widehat{y_t})$: the output-gap, which measures output relative to its natural or potential level (which would prevail if prices were flexible); (g_t): real consumption of public goods; (χ): share of private consumption in GDP.

Therefore, by combining equations (1) and (2), we obtain the following variation of global demand:

$$\widehat{y_t} = E_t(\widehat{y_{t+1}}) + (1-\chi)[\widehat{g_t} - E_t(\widehat{g_{t+1}})] - \chi[i_t - E_t(\pi_{t+1})] \tag{3}$$

This equation also implies that in the long term, the equilibrium nominal interest rate equals the inflation target: $(i = \pi^*)$.

Global demand is influenced by both monetary (the interest rate) and budgetary (public expenditure) policies. A higher interest rate increases savings and reduces consumption regarding households' decisions, and also reduces investment decisions. Therefore, a higher interest rate has recessionary consequences regarding the intertemporal choice of the representative household. On the contrary, higher public expenditure increases and sustains economic activity.

In the long run, public expenditure and economic activity are stable at their long-term equilibrium levels, $\lim_{n\to\infty}\widehat{y_n} = \lim_{n\to\infty}\widehat{g_n} = 0$; so by solving equation (3) forwards, we obtain:

$$\widehat{y_t} = -\chi\sum_{j=0}^{\infty}[i_{t+j} - E_t(\pi_{t+j+1})] + (1-\chi)\widehat{g_t} \tag{4}$$

Dolado *et al.* (2025) consider minimizing a quadratic per-period loss function in inflation and output performance, whereas Schaling (2024) considers inflation targeting where the central bank only aims at minimizing inflationary deviations. In our model, the monetary authority defines the nominal interest rate by minimizing a loss function depending on the respective weights given to the inflation (Φ_π^M) and economic activity (Φ_y^M) goals. Indeed, the central bank is committed to achieving its inflation objective, and the main goal is the inflation target (Φ_π^M is high); in particular, this is the main goal of the European Central Bank in Europe. However, the central bank can also consider economic activity (Φ_y^M). Besides, the central bank is constrained by the Zero Lower Bound: the nominal interest rate cannot decrease below zero ($i_t \geq 0$). Therefore, the nominal interest rate is fixed by the central bank according to a Taylor rule of the following form:

$$i_t = i + \Phi_\pi^M(\pi_t - \pi^*) + \Phi_y^M\widehat{y_t} \tag{5}$$

With: (π^*): central bank's inflation target.

By definition, the long-term and steady-state inflation rate is equal to the central bank's objective: $\lim_{n\to\infty}\pi_n = \pi^*$; the deviation of inflation from this target is: ($\widehat{\pi_t} = \pi_t - \pi^*$).

The variation of public expenditure ($\widehat{g_t}$) is fixed by the government, the budgetary authority, according to its relative preferences regarding economic activity and inflation. It also depends on the variation of the budgetary deficit allowed by fiscal rules and the previous trend of the public indebtedness of the government.

3.2. Inflation rate and nonlinear Phillips curve

To introduce non-linearities in the Phillips curve, Cristini and Ferri (2021) consider the log of the ratio between the rate of unemployment and the natural rate; they also introduce a dummy to consider that the slope can vary between positive and negative unemployment gaps. Harding et al. (2023) consider a quasi-kinked demand function of the following form: $Y_t = a - P_t^b, with\ a > 0, b > 0$, where (Pt) is the price level.

In this paper, we consider a traditional modeling of the variation of the inflation rate, used in DSGE models, relying on a New Keynesian Phillips Curve [see for example Eser et al. (2020), Benigno and Eggertsson (2023)]. A higher output-gap (or smaller unemployment rate) reduces efficiency and increases marginal production costs. It increases labor demand by firms, a demand that households are only willing to provide for higher wages, leading to cost increases for firms. However, in conformity with empirical results mentioned in section 2, we introduce non-linearity and convexity in this Phillips curve. We consider a hybrid Phillips curve, with a traditional backward-looking component of past inflation, but where the differential in inflation also depends on future expected inflation (as in New-Keynesian models), with a persistence of inflation (γ)[1]. So, the Phillips curve is as follows:

[1] On the contrary, in Schaling (2024), the non-linear Phillips curve is simply: $(\widehat{\pi_{t+1}} - \widehat{\pi_t}) = \frac{\alpha\widehat{y_t}}{(1-\alpha\varphi\widehat{y_t})}$. In Dolado *et al.* (2025), non-linearity is introduced with a quadratic function: $(\widehat{\pi_{t+1}} - \widehat{\pi_t}) = \alpha(\widehat{y_t} + \Phi\widehat{y_t}^2)$.

$$\left(\widehat{\pi_t}^{GDP} - \gamma\widehat{\pi_{t-1}}^{GDP}\right) = \frac{\kappa\widehat{y_t}}{(1-\kappa\varphi\widehat{y_t})} + \beta\left[E_t\left(\widehat{\pi_{t+1}}^{GDP}\right) - \gamma\widehat{\pi_t}^{GDP}\right] + s_t \quad (6)$$

With: $(\widehat{\pi_t}^{GDP})$: deviation of producer inflation (GDP deflator) from its long-term steady-state value; (s_t): innovations on desired mark-ups, related to the pricing decisions of firms, to the evolution of their margins. A positive shock to the mark-up corresponds to an increase whereas a negative shock corresponds to a decrease in profits.

(γ): persistence of inflation.

(β): time discount factor, discount rate of the future.

(κ>0): slope of the Phillips curve at the origin. Indeed: $\frac{\partial\left(\widehat{\pi_t}^{GDP} - \gamma\widehat{\pi_{t-1}}^{GDP}\right)}{\partial\widehat{y_t}} \xrightarrow[\widehat{y_t}\to 0]{} \kappa$. This slope is the arbitrage a country can do between inflation and economic growth in a broader context.

$(0 \le \varphi \le 1)$ is the curvature of the Phillips curve. Indeed, $\left(\widehat{\pi_t}^{GDP} - \gamma\widehat{\pi_{t-1}}^{GDP}\right) \xrightarrow[\varphi\to 0]{} \kappa\widehat{y_t}$ and the curve is then linear.

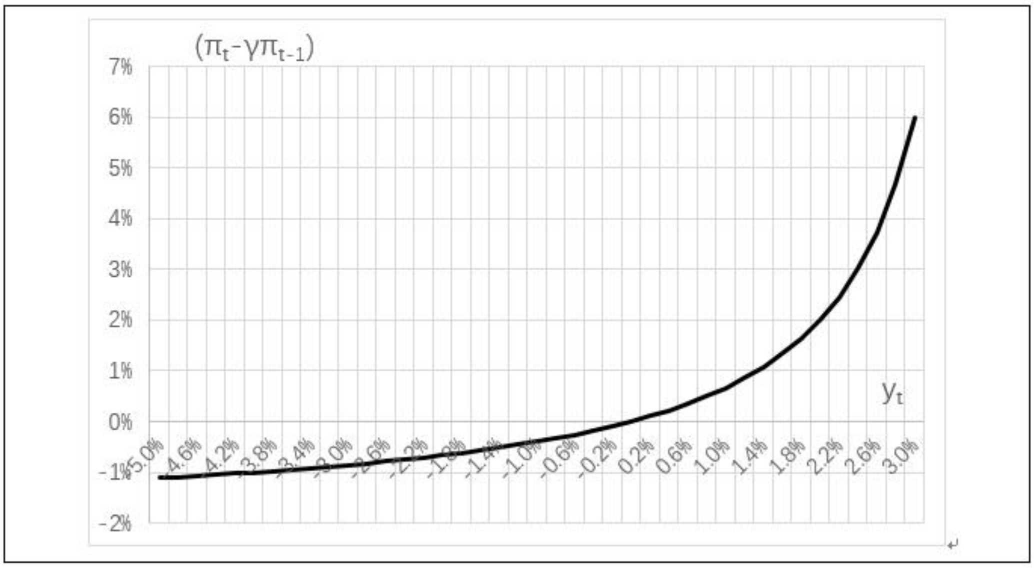

Figure 4. Phillips Curve: inflation variation according to the variation of economic activity.

Note: Calibration: (φ=0.5), (κ=0.5), (ν=0).

Therefore, Figure 4 represents the variation of inflation according to the variation of economic activity (Philips curve). And it is worth mentioning the main features of this non-linear Phillips curve.

- $\left(\widehat{\pi_t}^{GDP} - \gamma\widehat{\pi_{t-1}}^{GDP}\right) \xrightarrow[\widehat{y_t}\to\frac{1}{\kappa\varphi}]{} +\infty$: The inflation rate explodes for high growth rates, if the output-gap approaches $(\hat{y}^{max} = \frac{1}{\kappa\varphi})$, the upper bound of potential output in the short run.
- $\left(\widehat{\pi_t}^{GDP} - \gamma\widehat{\pi_{t-1}}^{GDP}\right) \xrightarrow[\widehat{y_t}\to-\infty]{} \hat{\pi}^{min} = -\frac{1}{\varphi}$: The inflation rate cannot go down a minimal level for weak growth rates.
- This non-linear Phillips curve is convex[2].

[2] Indeed, $\frac{\partial(\widehat{\pi_t}^{GDP} - \gamma\widehat{\pi_{t-1}}^{GDP})}{\partial\widehat{y_t}} = \frac{\kappa}{(1-\kappa\varphi\widehat{y_t})^2} > 0$ and: $\frac{\partial^2(\widehat{\pi_t}^{GDP} - \gamma\widehat{\pi_{t-1}}^{GDP})}{\partial^2\widehat{y_t}} = \frac{2\kappa^2\varphi}{(1-\kappa\varphi\widehat{y_t})^3} > 0$.

With: $E_t\left(\widehat{\pi_j}^{GDP}\right) \underset{j\to\infty}{\longrightarrow} 0$, iterating equation (6) forwards implies:

$$\widehat{\pi_t}^{GDP} = \gamma\widehat{\pi_{t-1}}^{GDP} + \sum_{j=0}^{\infty}\beta^j E_t\left(\frac{\kappa\widehat{y_{t+j}}}{\left(1-\kappa\varphi\widehat{y_{t+j}}\right)}\right) + \sum_{j=0}^{\infty}\beta^j E_t\left(s_{t+j}\right) \tag{7}$$

Furthermore, the deviation of consumer-prices inflation from its long-term steady-state value ($\widehat{\pi_t}$) increases with domestic producer prices, but also with import prices. Therefore, we have:

$$\widehat{\pi_t} = (1-\nu)\widehat{\pi_t}^{GDP} + \nu\widehat{\pi_t}^{M} \tag{8}$$

($\widehat{\pi_t}$): deviation of consumer inflation from its long-term steady-state value.
($\widehat{\pi_t}^{M}$): deviation of foreign prices from their long-term steady-state value.
(ν): openness rate of the country, share of imported goods in consumption.
Therefore, by combining equations (7) and (8), we obtain:

$$\widehat{\pi_t} = \gamma(1-\nu)\widehat{\pi_{t-1}}^{GDP} + \nu\widehat{\pi_t}^{M} + \sum_{j=0}^{\infty}\beta^j E_t\left(\frac{\kappa(1-\nu)\widehat{y_{t+j}}}{\left(1-\kappa\varphi\widehat{y_{t+j}}\right)}\right) + (1-\nu)\sum_{j=0}^{\infty}\beta^j E_t\left(s_{t+j}\right) \tag{9}$$

Current inflation is influenced by past inflation (path dependency and inertia) and foreign and imported inflation. It also depends on the future evolution path of the anticipated economic growth and variation of profit margins.

3.3. Calibration

In the economic literature mentioned by Furlanetto and Lepetit (2024), the slope of the Phillips curve varies between (-0.6) in 1974 and (-0.25) post-2000. As in Schaling (2004), we calibrate the slope of the Phillips curve at the origin at (κ=0.5), and the curvature of the Phillips curve at (φ=0.5). This average curvature introduces the non-linear response of economic variables observed in empirical data, particularly during the inflationary period following the war in Ukraine in 2021-2023 (see sections 1 and 2). According to World Bank Data, in 2023, the share of imported goods and services in GDP ranged from 13.9% in Argentina, 15.7% in Brazil, 17.6% in China, 22.4% in Australia, 32.5% in Italy, 33.8% in Canada, 34.1% in Spain, 36.3% in France, 39.4% in Germany, 57.3% in Austria, 77.4% in the Netherlands, 84.8% in Belgium, and even 181.7% in Luxembourg. We calibrate the openness rate at (ν=0.4), an average value for the European Union countries, for example, even if openness degrees are very heterogeneous. In 2023, the share of private consumption in GDP ranged from 0.39 in China, 0.5 in Australia, Germany, or Belgium, 0.53 in France, 0.55 in Canada, 0.58 in Italy or Poland, 0.62 in Portugal, 0.67 in Greece, and 0.68 in the United States. So, we calibrate this share at ($\chi = 0.6$).

Regarding the central bank's preferences, we consider a Taylor rule where price stability is the central bank's main goal ($\Phi_\pi^M = 1.5$). However, the central bank can also be concerned with economic activity ($\Phi_y^M = 1$). Empirically, the unique goal of the European Central Bank is to preserve price stability ($\Phi_y^M \to 0$). However, in the United States, for example, the FED has been assigned more balanced goals, a dual mandate: maximum employment and price stability.

4. Consequences of nonlinearity on main economic variables

4.1. A non-linear nominal interest rate

Let's suppose that the central bank succeeds in anchoring anticipations of future variations of its nominal interest rate according to equation (10):

$$\sum_{j=1}^{\infty} E_t\left(i_{t+j}-i\right)=\sum_{j=0}^{\infty} E_t\left(\widehat{\pi_{t+j+1}}\right)+\frac{(1-\chi)}{\chi}\widehat{g_t} \tag{10}$$

In this case, we can obtain economic activity and inflation variations according to the monetary policy, according to the differential between the effective nominal interest rate and the equilibrium long term nominal interest rate. Indeed, equations (4), (9), and (10) imply:

$$\widehat{y_t}=-\chi(i_t-i) \tag{11}$$

$$\widehat{\pi_t}=Z_t-\frac{\kappa(1-\nu)\chi(i_t-i)}{[1+\kappa\varphi\chi(i_t-i)]} \tag{12}$$

$$Z_t=\gamma(1-\nu)\widehat{\pi_{t-1}}^{GDP}+\nu\widehat{\pi_t}^{M}-\sum_{j=1}^{\infty}\beta^j E_t\left[\frac{\kappa(1-\nu)\chi\left(i_{t+j}-i\right)}{\left(1+\kappa\varphi\chi\left(i_{t+j}-i\right)\right)}\right]+(1-\nu)\sum_{j=0}^{\infty}\beta^j E_t\left(s_{t+j}\right)$$

Then, equations (5), (11), and (12) imply that for the central bank, the optimal variation of the nominal interest is not straightforward. Indeed, it could have two solutions[3]:

$$(i_{t1}-i)=\frac{\left[\kappa\varphi\chi\Phi_\pi^M Z_t-\left(1+\Phi_y^M\chi\right)-\Phi_\pi^M(1-\nu)\kappa\chi-\sqrt{X_t}\right]}{2\kappa\varphi\chi\left(1+\chi\Phi_y^M\right)}$$

$$(i_{t2}-i)=\frac{\left[\kappa\varphi\chi\Phi_\pi^M Z_t-\left(1+\Phi_y^M\chi\right)-\Phi_\pi^M(1-\nu)\kappa\chi+\sqrt{X_t}\right]}{2\kappa\varphi\chi\left(1+\chi\Phi_y^M\right)} \tag{13}$$

With[4]: $X_t=[\left(1+\Phi_y^M\chi\right)+\Phi_\pi^M(1-\nu-\varphi Z_t)\kappa\chi]^2+4\kappa\varphi\chi\left(1+\chi\Phi_y^M\right)\Phi_\pi^M Z_t$

Therefore, monetary policy has two solutions in equation (13): (i_{t1}) and (i_{t2}). However, $(i_{t1}-i)$ is always negative. Besides, $[(i_{t1}-i)\underset{\varphi\to 0}{\longrightarrow}-\infty]$. So, the solution (i_{t1}) could often be confronted with the Zero Lower Bound constraint for the variation of the nominal interest rate, as soon as the Phillips curve approaches linearity and as its curvature is weak. Therefore, we can consider that the preferred non-linear solution of our system is the nominal interest rate (i_{t2}). Indeed, this interest rate (i_{t2}) has a lower bound regarding the necessary decrease of the interest rate in the case of a recessionary economic conjuncture: $[(i_{t2}-i)\underset{Z_t\to-\infty}{\longrightarrow}-\frac{1}{\kappa\varphi\chi}$ and $(i_{t2}-i)\underset{Z_t\to+\infty}{\longrightarrow}+\infty]$, which is an advantage regarding the Zero Lower Bound constraint.

As in Schaling (2004), the nominal interest rate $(i_t=i_{t2})$ is then a non-linear and convex function of the variation of an inflationary supply shock represented by the variable (Zt) [see Figure 5]. Indeed, the interest rate response is asymmetric. In case of negative deviations from the inflation target $(Z_t<0)$, the decrease of the nominal interest rate remains limited, and there is a lower bound to its decrease $[-\frac{1}{\kappa\varphi\chi}<(i_t-i)<0$ according to equation

[3] $\kappa\varphi\chi\left(1+\chi\Phi_y^M\right)(i_t-i)^2-[\kappa\varphi\chi\Phi_\pi^M Z_t-\left(1+\Phi_y^M\chi\right)-\Phi_\pi^M(1-\nu)\kappa\chi](i_t-i)-\Phi_\pi^M Z_t=0.$

[4] Upper and lower bounds for $(\sqrt{X_t})$ according to the values of (Z_t) are given in Appendix A.1.

(13), and to the range of values for (X_t) in Appendix A.1]. With our basic calibration, this lower bound would correspond to a decrease of the nominal interest rate of 6.67% below the natural rate. If the economic conjuncture is stable $(Z_t = 0)$, the nominal interest rate can be fixed to its long-term equilibrium value $(i_t = i)$. However, in case of inflationary tensions $(Z_t > 0)$, the nominal interest rate can increase more than proportionally above its long-term equilibrium value $[0 < (i_t - i) < \frac{\Phi_\pi^M Z_t}{(1+\chi\Phi_y^M)}]$. So, positive deviations from the inflation target imply higher interest rate increases and penalties than negative deviations. Indeed, if inflationary pressures are high, the real interest rate is below its equilibrium level, which will increase the future output gap. So, the nonlinear Phillips curve necessitates a more contractionary monetary policy and an increase in interest rates to avoid these accentuated inflationary tensions. Dolado et al. (2005) also consider that the monetary authority, anticipating higher inflationary pressures if the Phillips curve is nonlinear, reacts more forcefully, as it has a greater incentive to avoid periods of excess demand, as these periods require longer and /or more severe recessions to fight against previous inflationary tensions.

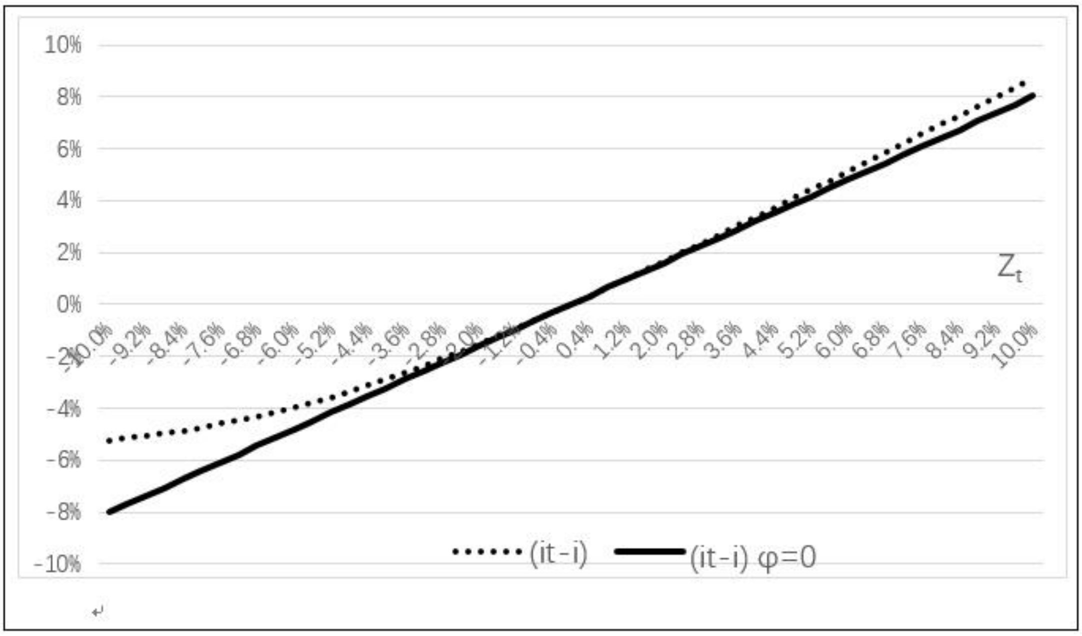

Figure 5. Variation of interest rate according to the economic conjuncture.

Note: Calibration: (φ=0.5), (κ=0.5), (ν=0.4), (χ=0.6), $(\Phi_\pi^M = 1.5)$, $(\Phi_y^M = 1)$. The variation of interest rate refers to the differential between the current interest rate and its long-term and equilibrium value (i). The solid line is the variation in the case of a linear Phillips curve, whereas the dotted line is for a non-linear Phillips curve.

Figure 5 underlines that the existence of a non-linear Phillips curve has smaller or larger implications according to the economic conjuncture. Indeed, in the case of a linear Phillips curve, the nominal interest rate should be as follows:

$$(i_t - i)_{(\varphi=0)} = \frac{\Phi_\pi^M}{\left[\left(1+\chi\Phi_y^M\right) + \Phi_\pi^M(1-\nu)\kappa\chi\right]} Z_t \quad (14)$$

However, if the Phillips curve is non-linear, the difference from this straight line is:

$$i_t - i_{t(\varphi=0)} = \frac{\Phi_\pi^M Z_t\left[\Phi_\pi^M(1-\nu)\kappa\chi - \left(1+\chi\Phi_y^M\right)\right]}{2\left(1+\chi\Phi_y^M\right)\left[\left(1+\chi\Phi_y^M\right) + \Phi_\pi^M(1-\nu)\kappa\chi\right]} - \frac{\left[\left(1+\Phi_y^M\chi\right) + \Phi_\pi^M(1-\nu)\kappa\chi - \sqrt{X_t}\right]}{2\kappa\varphi\chi\left(1+\chi\Phi_y^M\right)} > 0 \quad (15)$$

This differential is null if ($Z_t = 0$), but it is positive for all other values of deflationary or inflationary tensions, whatever the calibration of our parameters (see Appendix A.2). The short-term nominal interest rate is always higher under a non-linear Phillips curve than it would be under a linear curve. However, this differential is accentuated in the case of deflationary tensions ($Z_t < 0$), where the decrease of the nominal interest rate remains limited. Indeed, we obtain:

$$If\ Z_t \leq 0 \quad 0 \leq \left[i_t - i_{t(\varphi=0)}\right] < -\frac{\Phi_\pi^M Z_t}{\left[\left(1+\chi\Phi_y^M\right)+\Phi_\pi^M(1-\nu)\kappa\chi\right]} \tag{16}$$

On the contrary, this differential is more limited in the case of inflationary tensions ($Z_t > 0$), where the increase of the nominal interest rate can even be more than proportional to these inflationary tensions (see Figure 5). Indeed, we obtain:

$$If\ Z_t \geq 0 \quad 0 \leq \left[i_t - i_{t(\varphi=0)}\right] < \frac{\Phi_\pi^M(1-\nu)\kappa\chi}{\left(1+\chi\Phi_y^M\right)}\frac{\Phi_\pi^M Z_t}{\left[\left(1+\chi\Phi_y^M\right)+\Phi_\pi^M(1-\nu)\kappa\chi\right]} \tag{17}$$

This result is important regarding the empirical conduct of real-world central banking. Indeed, it means that monetary policy should respond asymmetrically to economic cycles and the conjuncture. In the case of recessionary or deflationary tensions, the decrease in the nominal interest rate should be limited. On the contrary, an active and efficient monetary policy of interest rate increases is much more necessary in the case of inflationary tensions, to allow the anchoring of private expectations.

Furthermore, a sensitivity analysis of the robustness of our results to the parameters of our model is necessary. As with a linear version of the Phillips curve [see equation (14)], the variation of interest rate in equation (13) is, obviously, an increasing function of the preference of the central bank for price stability [$\frac{(\partial i_t/\partial Z_t)}{\partial\Phi_\pi^M} > 0$], and a decreasing function of its preference for economic activity [$\frac{(\partial i_t/\partial Z_t)}{\partial\Phi_y^M} < 0$]. Besides, the slopes of the reaction functions in Figure 5 are more accentuated if the preference of the central bank for price stability is higher in comparison with its preference for stabilizing economic activity. Indeed, for example, empirically, Gelos and Ustyugova (2017) find that between 2001 and 2010, for 31 advanced and 61 emerging and developing economies, countries with more independent central banks and higher governance scores contained the impact of commodity price shocks better. However, our model also shows that the curvature of the non-linear reaction function in Figure 5 (the differential with the straight line and the linear case) increases with the relative preference of the central bank for price stability. Therefore, with a non-linear Phillips curve (see equation (15)), in case of inflationary tensions (Z_t is high), the risk of large increase in the nominal interest rate is still higher if the central bank has a higher and accentuated preference for stabilizing prices [(Φ_π^M/Φ_y^M) is high].

Besides, we can mention the sensitivity analysis of our results, regarding the specificity of an optimal monetary policy in the case of a convex Phillips curve, to various parameters of our model. According to equations (18) and (19), monetary policy can be less active if it is more efficient: if the slope of the Phillips curve at the origin is high (κ is high) or if the share of private consumption in GDP is high (χ is high). Therefore, in case of a deflationary shock (Z_t<0), monetary policy can be less expansionary: the interest rate decreases less and is higher. On the contrary, in case of an inflationary shock (Z_t>0), monetary policy can be less contractionary if it is more efficient: the nominal interest rate increases less. Indeed:

$$with\ \frac{\partial Z_t}{\partial\chi} = 0: \quad \frac{\partial i_t}{\partial\chi} = -\frac{\left[\Phi_\pi^M\kappa\chi(1-\nu+\varphi Z_t)+\left(1+\Phi_y^M\chi\right)-\sqrt{X_t}\right]}{2\kappa\varphi\chi^2\left(1+\chi\Phi_y^M\right)\sqrt{X_t}} \leq 0\ iif\ Z_t \geq 0 \tag{18}$$

$$with\ \frac{\partial Z_t}{\partial \kappa} = 0:\ \frac{\partial i_t}{\partial \kappa} = -\frac{\left[\Phi_\pi^M \kappa\chi(1-\nu+\varphi Z_t) + \left(1+\Phi_y^M \chi\right) - \sqrt{X_t}\right]}{2\kappa^2 \varphi\chi\sqrt{X_t}} \leq 0\ iif\ Z_t \geq 0 \tag{19}$$

Regarding the effect of the curvature of the Phillips curve, we obtain, with: $\frac{\partial Z_t}{\partial \varphi} = 0$:

$$\frac{\partial i_t}{\partial \varphi} = \frac{\Phi_\pi^M (1-\nu)\left[\Phi_\pi^M \kappa\chi(\varphi Z_t - 1 + \nu) + \sqrt{X_t}\right]}{2\varphi^2\left(1+\chi\Phi_y^M\right)\sqrt{X_t}} + \frac{\left[\sqrt{X_t} - \left(1+\Phi_y^M \chi\right) - \Phi_\pi^M (2-2\nu+\varphi Z_t)\kappa\chi\right]}{2\varphi^2 \kappa\chi\sqrt{X_t}} \tag{20}$$

So, according to Appendix A.3, with a non-linear Phillips curve, in the case of inflationary tensions ($Z_t > 0$), the higher the curvature of the curve (φ), the higher the nominal interest rate ($\frac{\partial i_t}{\partial \varphi} > 0$). Whereas the slope of the linear function is not affected, the curvature of the non-linear reaction function in Figure 5 (the differential with the straight line and the linear case) increases with (φ). Therefore, with a non-linear Phillips curve (see equation (15)), in case of inflationary tensions (Z_t is high), the risk of a large increase in the nominal interest rate is higher if the curvature of the Phillips curve (φ) is high.

Besides, a higher share of public spending in GDP (1-χ) or a weaker slope of the Phillips curve at the origin (κ) increases the slope of both curves in Figure 5, and the increase in the nominal interest rate with inflationary tensions. However, the effect of the non-linearity of the Phillips curve then becomes negligible (both curves in Figure 5 are nearly identical if χ or κ is small). A higher openness rate (ν) also slightly increases the slope of the reaction functions in Figure 5 without accentuating the curvature and the danger related to a non-linear Phillips curve.

4.2. Economic activity, private and public consumption

In case of deflationary tensions, the decrease of the nominal interest rate increases economic activity, whereas in the case of inflationary tensions, the increase of the nominal interest rate is harmful to economic growth. Besides, by combining equations (11) and (13) or (14), we can obtain the following discrepancy between the variation of economic activity in the case of a non-linear (21) or linear (22) Phillips curve:

$$\widehat{y_t} = -\frac{\left[\kappa\varphi\chi\Phi_\pi^M Z_t - \left(1+\Phi_y^M \chi\right) - \Phi_\pi^M (1-\nu)\kappa\chi + \sqrt{X_t}\right]}{2\kappa\varphi\left(1+\chi\Phi_y^M\right)} \tag{21}$$

$$\widehat{y_{t(\varphi=0)}} = -\frac{\chi\Phi_\pi^M}{\left[\left(1+\Phi_y^M \chi\right) + \Phi_\pi^M (1-\nu)\kappa\chi\right]} Z_t \tag{22}$$

In the same way, equations (2) and (21) or (22) imply the following discrepancy between the variation of private consumption in the case of a non-linear (23) or linear (24) Phillips curve:

$$\widehat{c_t} = -\frac{\left[\kappa\varphi\chi\Phi_\pi^M Z_t - \left(1+\Phi_y^M \chi\right) - \Phi_\pi^M (1-\nu)\kappa\chi + \sqrt{X_t}\right]}{2\kappa\chi\varphi\left(1+\chi\Phi_y^M\right)} - \frac{(1-\chi)}{\chi}\widehat{g_t} \tag{23}$$

$$\widehat{c_{t(\varphi=0)}} = -\frac{\Phi_\pi^M}{\left[\left(1+\Phi_y^M \chi\right) + \Phi_\pi^M (1-\nu)\kappa\chi\right]} Z_t - \frac{(1-\chi)}{\chi}\widehat{g_t} \tag{24}$$

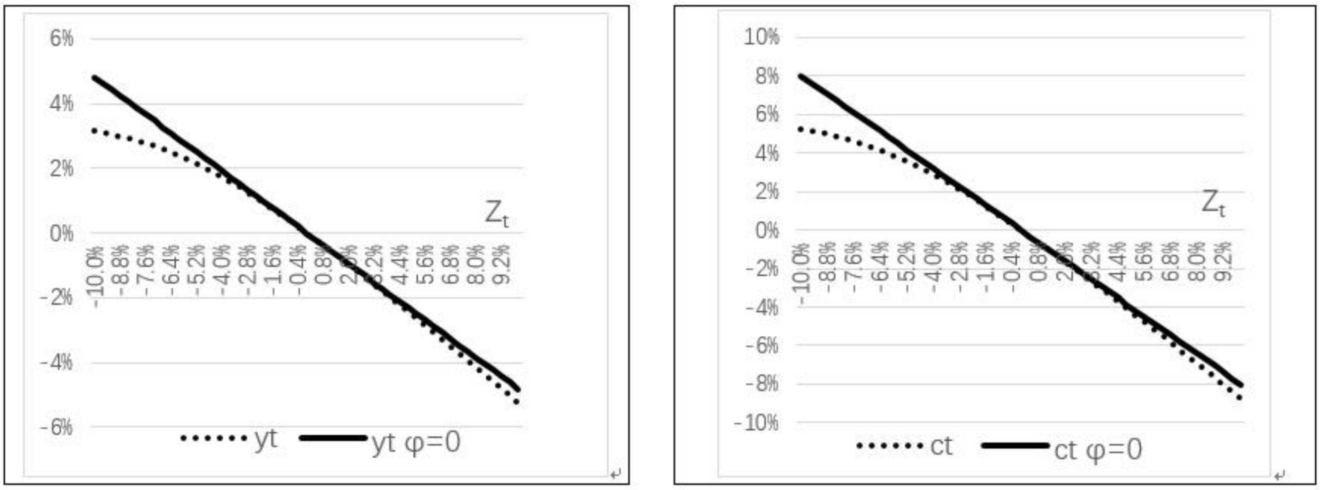

Figure 6. Variation of economic activity and private consumption according to the economic conjuncture.

Note: Calibration: (φ=0.5), (κ=0.5), (ν=0.4), (χ=0.6), ($\Phi_\pi^M = 1.5$), ($\Phi_y^M = 1$). The variation of economic activity (or private consumption) refers to the differential between the current economic activity (or private consumption) and its long-term and equilibrium value ($\widehat{y_t}, \widehat{c_t}$). The solid line is the variation in the case of a linear Phillips curve, whereas the dotted line is for a non-linear Phillips curve.

Therefore, economic growth ($\widehat{y_t}$) has an upper limit in the case of a recessionary economic conjuncture [see Figure 6]. Indeed, in the case of negative deviations from the inflation target ($Z_t < 0$), the decrease of the nominal interest rate remains limited, and there is an upper limit to economic growth [$0 < \widehat{y_t} < \frac{[(1+\Phi_y^M \chi)+\Phi_\pi^M(1-\nu)\kappa\chi]}{\kappa\varphi(1+\chi\Phi_y^M)}$ according to equation (21) and $0 < \widehat{c_t} < \frac{[(1+\Phi_y^M \chi)+\Phi_\pi^M(1-\nu)\kappa\chi]}{\varphi\kappa\chi(1+\chi\Phi_y^M)}$ according to equation (23) and to the range of values for (X_t) in Appendix A.1]. This upper limit would correspond to an increase of economic activity of 4.68%, and to a rise in private consumption of 7.79%, with our basic calibration. However, in the case of inflationary tensions ($Z_t > 0$), the economic recession can be accentuated by the large increase of the nominal interest rate [$-\frac{\chi\Phi_\pi^M Z_t}{(1+\chi\Phi_y^M)} < \widehat{y_t} < 0$ and $-\frac{\Phi_\pi^M Z_t}{(1+\chi\Phi_y^M)} < \widehat{c_t} < 0$]. This corresponds to a decrease of economic activity of ($-0.56Z_t < \widehat{y_t}$) and to a decrease of private consumption of ($-0.94Z_t < \widehat{c_t}$) according to our basic calibration.

Let's now turn to the sensitivity analysis of the robustness of our results to our model's parameters. The slopes of the reaction functions in Figure 6 are more accentuated if the central bank's preference for price stability is higher than its preference for stabilizing economic activity [(Φ_π^M/Φ_y^M) is high]. However, our model also shows that the curvature of the non-linear reaction function in Figure 6 (the differential with the straight line and the linear case) increases with the relative preference of the central bank for price stability. Therefore, in case of inflationary tensions (Z_t is high), with a non-linear Phillips curve, the economic recession would still be accentuated if the central bank has a higher and accentuated preference for stabilizing prices (see equations (21) and (23)). Indeed, monetary policy is then more recessionary.

Besides, the curvature of the non-linear reaction function in Figure 6 (the differential with the straight line and the linear case) increases with (φ). Therefore, with a non-linear Phillips curve (see equations (21) and (23)), in case of inflationary tensions (Z_t is high), recessionary risks are higher if the curvature of the Phillips curve (φ) is high. Furthermore, a higher share of public spending in GDP (1-χ) or a weaker slope of the Phillips curve at the origin (κ) increases the slopes of private consumption reaction functions in Figure 6, and the width of the potential recession with inflationary tensions. However, the effect of the non-linearity of the Phillips curve then becomes negligible (curves in Figure 6 are nearly identical if χ or κ is small). A higher openness rate (ν) also slightly increases the slope of the reaction functions in Figure 6 without accentuating the curvature and the danger related to non-linearity.

Furthermore, according to equation (23), in the case of inflationary tensions (Z_t is high), the more than proportional decrease in private consumption could only be avoided by a large increase in public expenditure ($\widehat{g_t^{**}}$), which could not be achieved without a more than proportional increase in public indebtedness level. Indeed, equations (2) and (21) imply:

$$\widehat{y_t} = 0 \quad iif: \quad \widehat{g_t^{**}} = \frac{\left[\kappa\varphi\chi\Phi_\pi^M Z_t - \left(1 + \Phi_y^M \chi\right) - \Phi_\pi^M (1-\nu)\kappa\chi + \sqrt{X_t}\right]}{2\kappa\varphi(1-\chi)\left(1+\chi\Phi_y^M\right)} \tag{25}$$

So, to stabilize economic activity, we obtain $[-\frac{[(1+\Phi_y^M\chi)+\Phi_\pi^M(1-\nu)\kappa\chi]}{\varphi\kappa(1+\chi\Phi_y^M)(1-\chi)} < \widehat{g_t^{**}} < 0]$ in the case of deflationary tensions ($Z_t < 0$), but $[0 < \widehat{g_t^{**}} < \frac{\chi\Phi_\pi^M Z_t}{(1-\chi)(1+\chi\Phi_y^M)}]$ in the case of inflationary tensions ($Z_t > 0$). The decrease of public expenditure is limited by ($\widehat{g_t^{**}} > -11.69\%$), whereas the increase of public spending can be more than proportional and quite unlimited ($\widehat{g_t^{**}} < 1.41Z_t$).

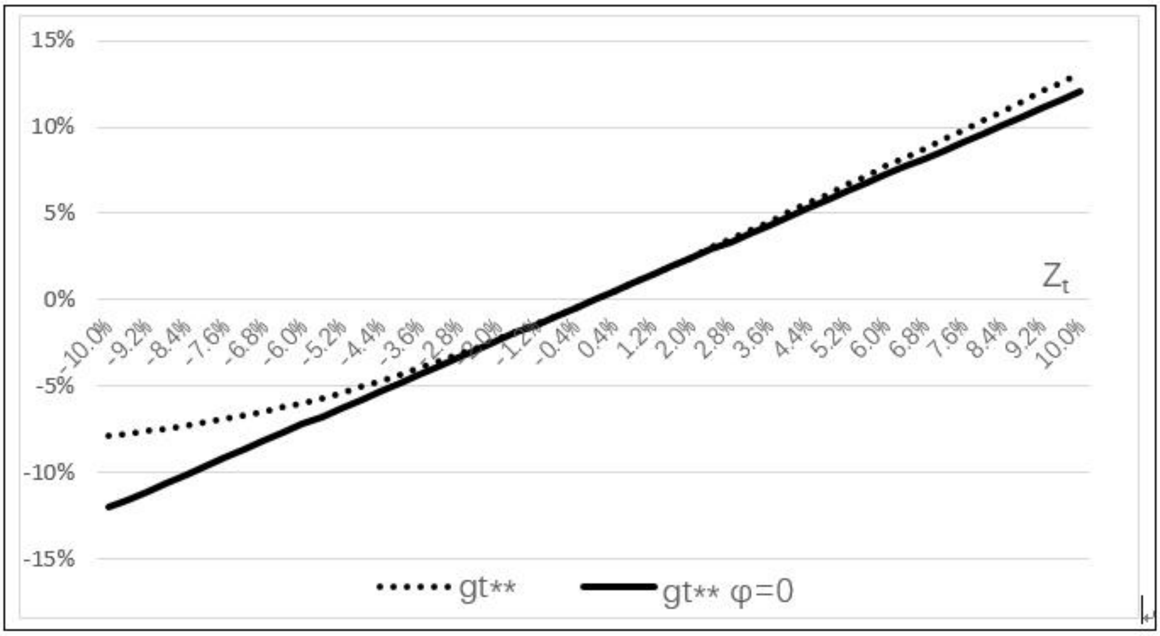

Figure 7. Variation of public expenditure necessary to stabilize economic activity.

Note: Calibration: (φ=0.5), (κ=0.5), (ν=0.4), (χ=0.6), ($\Phi_\pi^M = 1.5$), ($\Phi_y^M = 1$). ($\widehat{g_t^{**}}$) refers to the differential between the current public expenditure and its long-term and equilibrium value which would be necessary to stabilize economic activity ($\widehat{y_t} = 0$). The solid line is the variation in the case of a linear Phillips curve, whereas the dotted line is for a non-linear Phillips curve.

Therefore, our model highlights that with a non-linear Phillips curve, monetary policy can be less effective during inflationary tensions. Indeed, a decrease in the nominal interest rate can effectively support economic activity during recessionary tensions. Conversely, in the case of inflationary pressures, a restrictive monetary policy requires coordination with an active and expansionary fiscal policy (accentuated by a non-linear Phillips curve), which could pose risks to the long-term sustainability of public finances.

4.3. Inflation rate

Deflationary tensions ($Z_t < 0$) cannot fully be avoided by the decrease of the nominal interest rate, whereas inflationary tensions ($Z_t > 0$) cannot fully be avoided by the increase of the nominal interest rate. Indeed, by combining equations (12) and (13) or (14), we obtain the following discrepancy between inflation variation in the case of a non-linear (26) or linear (27) Phillips curve:

$$\widehat{\pi_t} = \frac{(\varphi Z_t - 1 + \nu)}{\varphi} + \frac{2(1 + \chi\Phi_y^M)(1 - \nu)}{\varphi[\kappa\chi\Phi_\pi^M(\varphi Z_t - 1 + \nu) + (1 + \chi\Phi_y^M) + \sqrt{X_t}]} \tag{26}$$

$$\widehat{\pi_{t(\varphi=0)}} = \frac{(1 + \Phi_y^M\chi)}{[(1 + \Phi_y^M\chi) + \Phi_\pi^M(1 - \nu)\kappa\chi]} Z_t \tag{27}$$

Therefore, deflationary tensions have a lower bound in the case of a recessionary economic conjuncture [see Figure 8]. Indeed, in the case of negative deviations from the inflation target ($Z_t < 0$), there is a lower bound to deflationary tensions [$-\frac{[(1+\Phi_y^M\chi)+\Phi_\pi^M(1-\nu)\kappa\chi]}{\Phi_\pi^M\varphi\kappa\chi} < \widehat{\pi_t} < 0$ according to Appendix A.4]. This lower bound would correspond to a decrease in prices of 8.31% with our basic calibration. However, in the case of an increase in prices, inflationary tensions ($Z_t > 0$) are more accentuated [$0 < \widehat{\pi_t} < Z_t$] and they are not limited by an upper limit if the Phillips curve is non-linear.

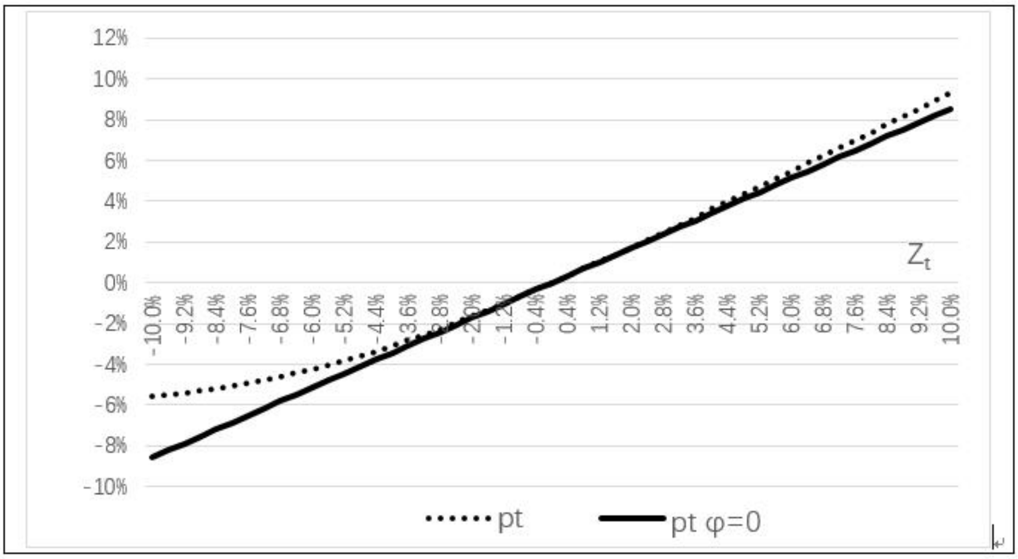

Figure 8. Variation of the inflation rate according to the economic conjuncture.

Note: Calibration: (φ=0.5), (κ=0.5), (ν=0.4), (χ=0.6), ($\Phi_\pi^M = 1.5$), ($\Phi_y^M = 1$). The variation of the inflation rate refers to the differential between the current inflation rate and its long-term and equilibrium value ($\widehat{\pi_t}$). The solid line is the variation in the case of a linear Phillips curve, whereas the dotted line is for a non-linear Phillips curve.

Let's now turn to the sensitivity analysis of the robustness of our results to our model's parameters. The slopes of the reaction functions in Figure 8 are more accentuated if the preference of the central bank for price stability is higher than its preference for stabilizing economic activity [(Φ_π^M/Φ_y^M) is high]. However, our model also shows that the curvature of the non-linear reaction function in Figure 8 (the differential with the straight line and the linear case) increases with the relative preference of the central bank for price stability. Therefore, in case of inflationary tensions (Z_t is high), with a non-linear Phillips curve, inflationary tensions would still be accentuated if the central bank has a higher and accentuated preference for stabilizing prices (see equation (26)), despite the more recessionary monetary policy.

Besides, the curvature of the non-linear reaction function in Figure 8 (the differential with the straight line and the linear case) increases with (φ). Therefore, with a non-linear Phillips curve (see equation (26)), in case of inflationary tensions (Z_t is high), recessionary risks are higher if the curvature of the Phillips curve (φ) is high. Furthermore, a higher share of public spending in GDP (1-χ) or a weaker slope of the Phillips curve at the origin (κ) increases the slopes of the reaction functions in Figure 8, and the potential inflationary tensions. However, the effect of the non-linearity of the Phillips curve then becomes negligible (curves in Figure 8 are nearly identical if χ or κ is small). A higher openness rate (ν) also slightly increases the slope of the reaction functions in Figure 8 without much accentuating the curvature and the danger related to non-linearity.

5. Conclusion

Our theoretical model can shed light on the difficulties encountered by central banks in fighting against inflationary tensions between 2021 and 2023. Indeed, empirical data show that the interest rate can mainly be a non-linear and convex function of the variation in an inflationary supply shock. The interest rate response is then asymmetric. In the case of negative deviations from the inflation target, the decrease in the nominal interest rate is limited by a lower bound. On the contrary, in the case of inflationary tensions, the nominal interest rate can increase more than proportionally above its long-term equilibrium value. The monetary authority, anticipating higher inflationary pressures if the Phillips curve is nonlinear, reacts more forcefully, as it has a greater incentive to avoid periods of excess demand, as these periods require longer and /or more severe recessions.

In this framework, we show that economic growth has an upper limit in the case of a recessionary economic conjuncture; on the contrary, in the case of inflationary tensions, the economic recession can be accentuated by the more than proportional increase in the nominal interest rate. The more than proportional decrease in private consumption could only be avoided by a large increase in public spending, which could not be achieved without a more than proportional increase in the public indebtedness level. In the case of a recessionary economic conjuncture, the decrease in the nominal interest rate cannot avoid deflationary tensions, but these tensions have a lower bound. However, in case of an increase in prices, inflationary tensions are more accentuated; despite the increase in the nominal interest rate, they are not limited by an upper limit if the Phillips curve is non-linear. The novelty of our model is to underline analytically the width of this asymmetric response, and to evaluate the width of the consequences related to a non-linear Phillips curve on private consumption and economic activity, thanks to the detailed analytical resolution of the model and to the sensitivity analysis of our results to various parameters.

These theoretical results can help explain why central banks had so many difficulties fighting against and limiting the huge worldwide inflationary tensions in 2021 and 2022, due to the resumption of demand after the COVID crisis and the war in Ukraine. Our results are useful to economic authorities, as they highlight why fighting against inflationary tensions can have a very high and more than proportional price to pay in terms of recession and unemployment. Non-linear Phillips curves appear to reveal specific challenges of monetary policy. In the case of recessionary tensions, the interest rate decrease should remain limited, and other policy tools such as non-conventional monetary policies could be useful. On the contrary, in the case of inflationary tensions, a large nominal interest rate increase appears necessary. Then, to limit the high price to pay of a recessionary monetary policy, the fine-tuning of this monetary policy is still more important. Good communication on the reasons for interest rate increases, and anchoring expectations to limit excessive inflationary expectations are probably still more necessary than in a cruising regime.

Funding Statement

This research received no external funding.

Acknowledgments

Acknowledgments to anonymous referees' comments and editor's effort.

Conflict of interest

The author claims that the manuscript is completely original. The author also declares no conflict of interest.

Author contributions

Conceptualization: Séverine Menguy; Investigation: Séverine Menguy; Methodology: Séverine Menguy; Formal analysis: Séverine Menguy; Writing – original draft: Séverine Menguy; Writing – review & editing: Séverine Menguy.

Appendix

A1. Values of X_t.

$$X_t = [(1+\Phi_y^M\chi) + \Phi_\pi^M(1-\nu-\varphi Z_t)\kappa\chi]^2 + 4\kappa\varphi\chi(1+\chi\Phi_y^M)\Phi_\pi^M Z_t$$
$$= [(1+\Phi_y^M\chi) - \Phi_\pi^M(1-\nu-\varphi Z_t)\kappa\chi]^2 + 4(1+\Phi_y^M\chi)\Phi_\pi^M(1-\nu)\kappa\chi$$
$$= \left[(1+\Phi_y^M\chi) + \Phi_\pi^M(1-\nu+\varphi Z_t)\kappa\chi\right]^2 - 4\Phi_\pi^{M2}(1-\nu)(\kappa\chi)^2\varphi Z_t$$

- If: $Z_t < -\frac{[(1+\Phi_y^M\chi)+\Phi_\pi^M(1-\nu)\kappa\chi]}{\Phi_\pi^M\varphi\kappa\chi} < 0$

$$0 < \Phi_\pi^M(-1+\nu-\varphi Z_t)\kappa\chi - (1+\Phi_y^M\chi) < \Phi_\pi^M(1-\nu-\varphi Z_t)\kappa\chi - (1+\Phi_y^M\chi) < \sqrt{X_t}$$
$$< (1+\Phi_y^M\chi) + \Phi_\pi^M(1-\nu-\varphi Z_t)\kappa\chi$$

- If: $-\frac{[(1+\Phi_y^M\chi)+\Phi_\pi^M(1-\nu)\kappa\chi]}{\Phi_\pi^M\varphi\kappa\chi} < Z_t \leq 0$

$$0 < (1+\Phi_y^M\chi) + \Phi_\pi^M(1-\nu+\varphi Z_t)\kappa\chi < \sqrt{X_t} < (1+\Phi_y^M\chi) + \Phi_\pi^M(1-\nu-\varphi Z_t)\kappa\chi$$

- If: $0 \leq Z_t < \frac{[\Phi_\pi^M(1-\nu)\kappa\chi-(1+\Phi_y^M\chi)]}{\Phi_\pi^M\varphi\kappa\chi}$

$$0 < \Phi_\pi^M(1-\nu-\varphi Z_t)\kappa\chi - (1+\Phi_y^M\chi) < \Phi_\pi^M(1-\nu-\varphi Z_t)\kappa\chi + (1+\Phi_y^M\chi) < \sqrt{X_t}$$
$$< (1+\Phi_y^M\chi) + \Phi_\pi^M(1-\nu+\varphi Z_t)\kappa\chi$$

- If: $\frac{[\Phi_\pi^M(1-\nu)\kappa\chi-(1+\Phi_y^M\chi)]}{\Phi_\pi^M\varphi\kappa\chi} < Z_t < \frac{[\Phi_\pi^M(1-\nu)\kappa\chi+(1+\Phi_y^M\chi)]}{\Phi_\pi^M\varphi\kappa\chi}$

$$0 < (1+\Phi_y^M\chi) - \Phi_\pi^M(\varphi Z_t-1+\nu)\kappa\chi < (1+\Phi_y^M\chi) + \Phi_\pi^M(\varphi Z_t-1+\nu)\kappa\chi < \sqrt{X_t}$$
$$< (1+\Phi_y^M\chi) + \Phi_\pi^M(1-\nu+\varphi Z_t)\kappa\chi$$

- If: $0 < \frac{[\Phi_\pi^M(1-\nu)\kappa\chi+(1+\Phi_y^M\chi)]}{\Phi_\pi^M\varphi\kappa\chi} < Z_t$

$$0 < \Phi_\pi^M(\varphi Z_t-1+\nu)\kappa\chi - (1+\Phi_y^M\chi) < \Phi_\pi^M(\varphi Z_t-1+\nu)\kappa\chi + (1+\Phi_y^M\chi) < \sqrt{X_t}$$
$$< (1+\Phi_y^M\chi) + \Phi_\pi^M(1-\nu+\varphi Z_t)\kappa\chi$$

A2. Excess of the interest rate if the Phillips curve is non-linear.

- If: $Z_t < -\frac{[(1+\Phi_y^M\chi)+\Phi_\pi^M(1-\nu)\kappa\chi]}{\Phi_\pi^M\varphi\kappa\chi} < 0$

$$0 < \frac{[\Phi_\pi^M(-1+\nu-\varphi Z_t)\kappa\chi - (1+\chi\Phi_y^M)]}{\kappa\varphi\chi[(1+\chi\Phi_y^M) + \Phi_\pi^M(1-\nu)\kappa\chi]} < \left[i_t - i_{t(\varphi=0)}\right] < -\frac{\Phi_\pi^M Z_t}{[(1+\chi\Phi_y^M) + \Phi_\pi^M(1-\nu)\kappa\chi]}$$

- If: $-\frac{[(1+\Phi_y^M\chi)+\Phi_\pi^M(1-\nu)\kappa\chi]}{\Phi_\pi^M\varphi\kappa\chi} < Z_t < 0$

$$\frac{\Phi_\pi^{M2}(1-\nu)\kappa\chi Z_t}{(1+\chi\Phi_y^M)[(1+\chi\Phi_y^M)+\Phi_\pi^M(1-\nu)\kappa\chi]} < 0 < \left[i_t - i_{t(\varphi=0)}\right] < -\frac{\Phi_\pi^M Z_t}{[(1+\chi\Phi_y^M)+\Phi_\pi^M(1-\nu)\kappa\chi]} < \frac{1}{\varphi\kappa\chi}$$

- If: $0 < Z_t < \frac{[\Phi_\pi^M(1-\nu)\kappa\chi+(1+\Phi_y^M\chi)]}{\Phi_\pi^M\varphi\kappa\chi}$

$$0 < \left[i_t - i_{t(\varphi=0)}\right] < \frac{\Phi_\pi^{M2}(1-\nu)\kappa\chi Z_t}{(1+\chi\Phi_y^M)[(1+\chi\Phi_y^M)+\Phi_\pi^M(1-\nu)\kappa\chi]} < \frac{\Phi_\pi^M(1-\nu)}{\varphi(1+\chi\Phi_y^M)}$$

- If: $0 < \frac{[\Phi_\pi^M(1-\nu)\kappa\chi+(1+\Phi_y^M\chi)]}{\Phi_\pi^M\varphi\kappa\chi} < Z_t$

$$0 < \frac{\Phi_\pi^M(1-\nu)[\Phi_\pi^M\kappa\chi(\varphi Z_t - 1 + \nu) - (1+\chi\Phi_y^M)]}{\varphi(1+\chi\Phi_y^M)[(1+\chi\Phi_y^M)+\Phi_\pi^M(1-\nu)\kappa\chi]} < [i_t - i_{t(\varphi=0)}] < \frac{\Phi_\pi^{M2}(1-\nu)\kappa\chi Z_t}{(1+\chi\Phi_y^M)[(1+\chi\Phi_y^M)+\Phi_\pi^M(1-\nu)\kappa\chi]}$$

A3. Interest rate and curvature of the Phillips curve.

- If: $0 < Z_t < \frac{[\Phi_\pi^M(1-\nu)\kappa\chi-(1+\Phi_y^M\chi)]}{\Phi_\pi^M\varphi\kappa\chi} < \frac{[\Phi_\pi^M(1-\nu)\kappa\chi+(1+\Phi_y^M\chi)]}{\Phi_\pi^M\varphi\kappa\chi}$

$$0 < \frac{\Phi_\pi^M[\Phi_\pi^M\kappa\chi(1-\nu)+(1+\Phi_y^M\chi)][\Phi_\pi^M\kappa\chi(1-\nu-\varphi Z_t)-(1+\Phi_y^M\chi)]Z_t}{\varphi(1+\chi\Phi_y^M)[\Phi_\pi^M(1-\nu+\varphi Z_t)\kappa\chi+(1+\Phi_y^M\chi)][\Phi_\pi^M(1-\nu-\varphi Z_t)\kappa\chi+(1+\Phi_y^M\chi)]}$$
$$-\frac{\Phi_\pi^{M2}\kappa\chi[\Phi_\pi^M(1-\nu)^2\kappa\chi+(1-\nu-\varphi Z_t)(1+\Phi_y^M\chi)]Z_t}{\varphi(1+\chi\Phi_y^M)[\Phi_\pi^M(1-\nu+\varphi Z_t)\kappa\chi+(1+\Phi_y^M\chi)][\Phi_\pi^M(1-\nu-\varphi Z_t)\kappa\chi+(1+\Phi_y^M\chi)]}$$
$$< \frac{\partial i_t}{\partial\varphi} < \frac{\Phi_\pi^{M2}\kappa\chi[\Phi_\pi^M(1-\nu)^2\kappa\chi+(1-\nu-\varphi Z_t)(1+\Phi_y^M\chi)]Z_t}{\varphi(1+\chi\Phi_y^M)[\Phi_\pi^M(1-\nu+\varphi Z_t)\kappa\chi+(1+\Phi_y^M\chi)][\Phi_\pi^M(1-\nu-\varphi Z_t)\kappa\chi+(1+\Phi_y^M\chi)]}$$

- If : $0 < \frac{[\Phi_\pi^M(1-\nu)\kappa\chi-(1+\Phi_y^M\chi)]}{\Phi_\pi^M\varphi\kappa\chi} < Z_t < \frac{[\Phi_\pi^M(1-\nu)\kappa\chi+(1+\Phi_y^M\chi)]}{\Phi_\pi^M\varphi\kappa\chi}$

Or: $\frac{[\Phi_\pi^M(1-\nu)\kappa\chi-(1+\Phi_y^M\chi)]}{\Phi_\pi^M\varphi\kappa\chi} < 0 < Z_t < \frac{[\Phi_\pi^M(1-\nu)\kappa\chi+(1+\Phi_y^M\chi)]}{\Phi_\pi^M\varphi\kappa\chi}$

$$-\frac{(1-\nu)[\Phi_\pi^M\kappa\chi(1-\nu-\varphi Z_t)+(1+\chi\Phi_y^M)]\Phi_\pi^M}{\varphi^2(1+\chi\Phi_y^M)[(1+\Phi_y^M\chi)+\Phi_\pi^M(\varphi Z_t-1+\nu)\kappa\chi]} \leq 0 < \frac{\partial i_t}{\partial\varphi}$$
$$< \frac{\Phi_\pi^{M2}\kappa\chi(1-\nu)Z_t}{\varphi(1+\chi\Phi_y^M)[(1+\Phi_y^M\chi)+\Phi_\pi^M(1-\nu+\varphi Z_t)\kappa\chi]}$$

- If: $0 < \frac{[\Phi_\pi^M(1-\nu)\kappa\chi-(1+\Phi_y^M\chi)]}{\Phi_\pi^M\varphi\kappa\chi} < \frac{[\Phi_\pi^M(1-\nu)\kappa\chi+(1+\Phi_y^M\chi)]}{\Phi_\pi^M\varphi\kappa\chi} < Z_t$

$$0 < -\frac{\Phi_\pi^{M2}(1-\nu)^2\kappa\chi[(1+\Phi_y^M\chi)+\Phi_\pi^M(2-2\nu+\varphi Z_t)\kappa\chi]}{\varphi^2(1+\chi\Phi_y^M)[(1+\Phi_y^M\chi)+\Phi_\pi^M(1-\nu+\varphi Z_t)\kappa\chi][\Phi_\pi^M(\varphi Z_t-1+\nu)\kappa\chi+(1+\Phi_y^M\chi)]}$$
$$+\frac{\Phi_\pi^M(1-\nu)[\Phi_\pi^M(\varphi Z_t-1+\nu)\kappa\chi-(1+\Phi_y^M\chi)]}{\varphi^2[(1+\Phi_y^M\chi)+\Phi_\pi^M(1-\nu+\varphi Z_t)\kappa\chi][\Phi_\pi^M(\varphi Z_t-1+\nu)\kappa\chi+(1+\Phi_y^M\chi)]} < \frac{\partial i_t}{\partial\varphi}$$
$$< \frac{\Phi_\pi^{M3}Z_t^2(\kappa\chi)^2(1-\nu)}{(1+\chi\Phi_y^M)[(1+\Phi_y^M\chi)+\Phi_\pi^M(1-\nu+\varphi Z_t)\kappa\chi][\Phi_\pi^M(\varphi Z_t-1+\nu)\kappa\chi+(1+\Phi_y^M\chi)]}$$

- If: $\frac{[\Phi_\pi^M(1-\nu)\kappa\chi-(1+\Phi_y^M\chi)]}{\Phi_\pi^M\varphi\kappa\chi} < 0 < \frac{[\Phi_\pi^M(1-\nu)\kappa\chi+(1+\Phi_y^M\chi)]}{\Phi_\pi^M\varphi\kappa\chi} < Z_t$

$$0 < \frac{\Phi_\pi^M(1-\nu)[\Phi_\pi^M\kappa\chi(\varphi Z_t-1+\nu)-(1+\chi\Phi_y^M)]}{\varphi^2(1+\chi\Phi_y^M)[\Phi_\pi^M(\varphi Z_t-1+\nu)\kappa\chi+(1+\Phi_y^M\chi)]} < \frac{\partial i_t}{\partial\varphi} < \frac{\Phi_\pi^{M2}Z_t\kappa\chi(1-\nu)}{\varphi(1+\chi\Phi_y^M)[(1+\Phi_y^M\chi)+\Phi_\pi^M(1-\nu+\varphi Z_t)\kappa\chi]}$$

A4. Non-linear Phillips curve and inflation rates.

According to the values for (X_t) in Appendix A.1, and to equation (26):

- If: $Z_t < -\frac{\left[\left(1+\Phi_y^M\chi\right)+\Phi_\pi^M(1-\nu)\kappa\chi\right]}{\Phi_\pi^M\varphi\kappa\chi} < 0$

$$0 < \frac{1}{2\varphi\left(1+\Phi_y^M\chi\right)} < \frac{1}{\varphi\left[\sqrt{X_t}+\kappa\varphi\chi\Phi_\pi^M Z_t+\left(1+\chi\Phi_y^M\right)-\Phi_\pi^M(1-\nu)\kappa\chi\right]}$$

$$(\varphi Z_t-1+\nu)\kappa\chi\Phi_\pi^M(\varphi Z_t-1+\nu)+(\varphi Z_t+1-\nu)\left(1+\chi\Phi_y^M\right)+(\varphi Z_t-1+\nu)\sqrt{X_t} < 2\varphi Z_t\left(1+\chi\Phi_y^M\right) < 0$$

And therefore: $\widehat{\pi_t} < 0 \quad \lim_{Z_t\to-\infty}\sqrt{X_t} = \Phi_\pi^M(1-\nu-\varphi Z_t)\kappa\chi-\left(1+\Phi_y^M\chi\right)$

$$\lim_{Z_t\to-\infty}\widehat{\pi_t} = -\frac{\left(1+\chi\Phi_y^M\right)(Z_t-Z_t)}{\varphi\kappa\chi\Phi_\pi^M(Z_t-Z_t)}+\frac{2\left(1+\chi\Phi_y^M\right)(1-\nu)}{\varphi^2\kappa\chi\Phi_\pi^M(Z_t-Z_t)} = -\frac{\left(1+\chi\Phi_y^M\right)}{\varphi\kappa\chi\Phi_\pi^M}$$

- If: $-\frac{\left[\left(1+\Phi_y^M\chi\right)+\Phi_\pi^M(1-\nu)\kappa\chi\right]}{\Phi_\pi^M\varphi\kappa\chi} < Z_t < 0$

$$0 < 2\varphi\left[\kappa\chi\Phi_\pi^M\varphi Z_t+\left(1+\chi\Phi_y^M\right)\right] < \varphi\left[\kappa\chi\Phi_\pi^M(\varphi Z_t-1+\nu)+\left(1+\chi\Phi_y^M\right)+\sqrt{X_t}\right] < 2\varphi\left(1+\chi\Phi_y^M\right)$$

$$-\frac{\left[\left(1+\Phi_y^M\chi\right)+\Phi_\pi^M(1-\nu)\kappa\chi\right]}{\Phi_\pi^M\varphi\kappa\chi} < Z_t < \widehat{\pi_t} < \frac{Z_t\left[\kappa\chi\Phi_\pi^M(\varphi Z_t-1+\nu)+\left(1+\chi\Phi_y^M\right)\right]}{\left[\kappa\chi\Phi_\pi^M\varphi Z_t+\left(1+\chi\Phi_y^M\right)\right]} < 0$$

- If: $0 < Z_t < \frac{\left[\Phi_\pi^M(1-\nu)\kappa\chi-\left(1+\Phi_y^M\chi\right)\right]}{\Phi_\pi^M\varphi\kappa\chi}$

$$\frac{\left[(\varphi Z_t-1+\nu)\kappa\chi\Phi_\pi^M+\left(1+\chi\Phi_y^M\right)\right]Z_t}{\left[\kappa\chi\Phi_\pi^M\varphi Z_t+\left(1+\chi\Phi_y^M\right)\right]} < 0 < \widehat{\pi_t} < Z_t < \frac{\left[\Phi_\pi^M(1-\nu)\kappa\chi+\left(1+\Phi_y^M\chi\right)\right]}{\Phi_\pi^M\varphi\kappa\chi}$$

- If: $\frac{\left[\Phi_\pi^M(1-\nu)\kappa\chi-\left(1+\Phi_y^M\chi\right)\right]}{\Phi_\pi^M\varphi\kappa\chi} < Z_t < \frac{\left[\Phi_\pi^M(1-\nu)\kappa\chi+\left(1+\Phi_y^M\chi\right)\right]}{\Phi_\pi^M\varphi\kappa\chi}$

$$0 < \frac{\left[(\varphi Z_t-1+\nu)\kappa\chi\Phi_\pi^M+\left(1+\chi\Phi_y^M\right)\right]Z_t}{\left[\kappa\chi\Phi_\pi^M\varphi Z_t+\left(1+\chi\Phi_y^M\right)\right]} < \widehat{\pi_t}$$

$$< Z_t-\frac{\kappa\chi\Phi_\pi^M(1-\nu)(\varphi Z_t-1+\nu)}{\varphi\left[\kappa\chi\Phi_\pi^M(\varphi Z_t-1+\nu)+\left(1+\chi\Phi_y^M\right)\right]} < Z_t < \frac{\left[\Phi_\pi^M(1-\nu)\kappa\chi+\left(1+\Phi_y^M\chi\right)\right]}{\Phi_\pi^M\varphi\kappa\chi}$$

- If: $0 < \frac{\left[\Phi_\pi^M(1-\nu)\kappa\chi+\left(1+\Phi_y^M\chi\right)\right]}{\Phi_\pi^M\varphi\kappa\chi} < Z_t$

$$0 < \frac{\left[\left(1+\Phi_y^M\chi\right)+(\varphi Z_t-1+\nu)\kappa\chi\Phi_\pi^M\right]Z_t}{\left[\left(1+\Phi_y^M\chi\right)+\kappa\varphi\chi\Phi_\pi^M Z_t\right]} < \widehat{\pi_t} < Z_t-\frac{(1-\nu)\Phi_\pi^M(\varphi Z_t-1+\nu)\kappa\chi}{\varphi\left[\Phi_\pi^M(\varphi Z_t-1+\nu)\kappa\chi+\left(1+\Phi_y^M\chi\right)\right]} < Z_t$$

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