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Fundamental character of the risk premium to influence the sustainability of the public debt

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ABSTRACT

Traditionally, conditions of sustainability of the public debt have long been related quite exclusively to fiscal policy and to budgetary parameters. However, the interaction between fiscal and monetary policies regarding the fixation of the interest rate is fundamental. Indeed, a simple analytical modelling shows that if the nominal interest rate increases exponentially with the public debt, because of a default (credit) risk premium, if the confidence of investors is fundamental, conditions of sustainability of the public debt could be much more difficult to comply with. Indeed, if the interest rate is risk-free, values for which the public debt can be sustainable are less constraining if the long-term GDP growth rate is high, or if the long-term risk-free nominal interest rate is small. They are also less constraining if the country decides to turn to a non-negligible primary budget surplus in case of a high public debt. However, if the interest rate exponentially increases with the public debt level, in case of a significant importance of the default (credit) risk premium, these parameters have very limited consequences on sustainable and equilibrium public debt levels. The sustainable public debt that a government should target is then much smaller than in absence of this risk premium.

KEYWORDS

Interest rate; Risk premium; Public debt; Debt sustainability

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1. Introduction

Conditions of sustainability of the public debt have been largely studied in the economic literature. They are traditionally less restrictive if the long-term growth rate is high and if the nominal interest rate is low, or at least for a sufficiently positive relation between both variables [see for example Bohn (1998)]. However, one fundamental parameter is also and mainly the sensibility of the interest rate to the public debt, which is sometimes not sufficiently underlined by economic analysis. Indeed, fiscal policy should not be studied alone in a closed circle, in order to analyze rigorously the conditions of sustainability of the public debt. The interaction with monetary policy and with the fixation of the interest rate is fundamental; fiscal and monetary policies should be studied jointly. In a dynamic framework, and if the confidence of risk-adverse investors is fundamental to contain the public debt, the sustainability of the public debt depends on the mix between budgetary and monetary policy, which interaction should be taken into account (Bischi et al., 2022).

Before 2022, the environment of ultra-low interest rates contributed to keep away the danger of excessively high public debt levels for their sustainability in many OECD countries. However, countries with high public debt levels often pay risk premia, corresponding to excess interest rate returns beyond the risk-free nominal interest rate on the bonds market. Since 2022, renewed inflationary tensions and interest rates reaching again levels unknown for a long time imply a reactivated risk for the sustainability of some countries' public debts. Afonso et al. (2015) study the determinants of long-term sovereign bond yield between 1999 and 2010, for ten Euro area countries. Among factors, they show that before the financial crisis, macro and fiscal fundamentals were not significant in explaining spreads. However, since March 2009, financial markets penalize fiscal imbalances more strongly, attaching an extra premium on the stock of projected public debt. In the same way, Delatte et al. (2017) underline the existence of regime switch in the spread determination model for Euro-area peripheral sovereigns. Indeed, after the financial crisis in 2007, they show that yields have become much more sensitive to fundamentals, because of the deterioration of the banks' risk and of liquidity spirals. Therefore, the current macro-economic situation implies that the question of the sustainability of the public debt is again at the center of the political and economic debates.

In this context, the current paper aims at studying the relation between the risk premium beyond the risk-free interest rate (because investors are risk adverse) and the sustainability of the public debt. Therefore, we complexify the theoretical equation of the variation of the public debt, which depends on the budget deficit, on the risk-free interest rate and on economic growth, by introducing two supplementary equations. First, the primary budget deficit itself can depend on the public debt level (fiscal discipline or 'fiscal fatigue') [see Gosh et al. (2013), Ostry et al. (2010), Medeiros (2012)]. Second, beyond the risk-free interest rate, the interest rate can include a credit (default) risk premium increasing with the public debt level [see: Fournier and Fall (2017) or De Grauwe and Ji (2013)]. We then derive theoretically interesting results regarding equilibrium public debt levels: which equilibrium is stable, and beyond which level the public debt risks to explode. We show that if the interest rate is risk-free, values for which the public debt can be sustainable are less constraining if the long-term GDP growth rate is high, or if the long-term nominal risk-free interest rate is weak. They are also less constraining if the government decides a higher budget surplus if public indebtedness becomes excessive. However, if the nominal interest rate increases exponentially with the public debt level, because of a default (credit) risk premium, these parameters have very limited consequences on sustainable and equilibrium public debt levels. So, if the confidence of investors is a fundamental parameter for the weight of reimbursement of the public debt, conditions for sustainability of the public debt could be much more difficult to comply with.

The rest of the paper is organized as follows. The second section recalls some results of the economic literature on the link between the risk premium beyond the risk-free interest rate and the sustainability of the public debt, and on the importance of the mix between monetary and fiscal policies for ensuring the solvability of the public

debt. The third section presents a simple macro-economic modelling of the variation of the public debt, taking into account 'fiscal fatigue' for the variation of the budget deficit according to the public debt, and the potential exponential variation of the interest rate with the public indebtedness level (risk premium). The fourth section studies conditions for public debt sustainability according to various parameters. The fifth section concludes the paper.

2. Economic literature

2.1. Multiplicity of the factors influencing the sustainability of the public debt

A large economic literature studies the conditions of sustainability of the public debt and defines numerical targets below which the public debt must be contained in order to remain sustainable. For example, Checherita-Westphal et al. (2014) highlight the importance of debt-related fiscal rules and derive growth-maximizing public debt ratios from a simple theoretical model. On the basis of evidence on the productivity of public capital, they estimate public debt targets that governments should try to maintain if they wish to maximize growth for panels of OECD, EU and euro area countries, respectively. These numbers are founded on long-run optimizing behavior, assuming that governments implement the so-called golden rule over the cycle: they contract debt only to finance public investment. According to the authors, the optimal public debt ratio for a country depends on the public capital to be financed, in comparison with the private capital ratio. Their estimates suggest that Euro area countries should target debt levels of around 50% of GDP if member states are to have common targets, whereas a target of 67% of GDP would be allowed for a sample of 22 OECD countries.

Furthermore, Aldama and Creel (2020) demonstrate that long-run (or global) fiscal sustainability not only depends on regime-specific feedback coefficients of the fiscal policy rule, but also on the average durations of fiscal regimes. It is possible that a fiscal regime be periodically unsustainable (violating the condition of Bohn (1998)) without being unsustainable in the long run. Fiscal policy can be locally unsustainable, with a periodically explosive public-debt-to-GDP ratio, and still be globally sustainable. For example, evidence on French data suggests that France has achieved global sustainability since 1965, despite a prolonged period of unsustainability from the early 80s to the mid-90s.

Andrès et al. (2020) develop a DSGE model of a two-country monetary union, calibrated to match the characteristics of Spain and Germany, in which debt sustainability is endogenously determined to shape the responses of the risk premium on public debt. They find that in normal times, the costs of a government spending driven fiscal consolidation in the high-debt country are greatly diminished when this consolidation improves endogenously its debt sustainability prospects. On the contrary, when monetary policy is constrained at the Zero Lower Bound (ZLB), fiscal consolidation generates deflation expectations which increase the real interest rate and may compensate the benefits from a lower risk premium. In this context, a fiscal expansion in the low-debt country and a consolidation in the high-debt country delivers the greatest positive impact on union-wide output.

There is a large theoretical as well as empirical literature about the sustainability of the public debt; beyond economic growth and the interest rate on the bonds market, many external and uncertain factors may influence this long-term sustainability. The goal if this paper is to shed light on the importance of the risk premium beyond the risk-free nominal interest rate on the public debt, and mainly on the default (credit) risk premium.

2.2. Importance of the default (credit) risk premium

The public debt can be considered as risky. If it is not sustainable, there could be a risk of devaluation, as a country is then tempted to reduce this way the weight of its public debt denominated in domestic currency. On the

contrary, if a devaluation is not possible, because of fixed exchange rates and monetary unification in particular, the country could have to default on its public debt. Therefore, as mentioned by Alesina et al. (1992), in Europe, monetary unification with an European Central Bank committed to low inflation eliminates inflation risk on government debts, but it may have increased the risk of solvency crisis for highly indebted governments. Indeed, the interest rate on the public debt incorporates a risk premium, linked to the estimated sustainability of the public debt, or to the potential risk of default on this public debt. If the public debt is anticipated as beyond a sustainable level, a sovereign default risk premium is applied on the interest rate, and there is then usually a non-linear relation between the public debt and the interest rate on this public debt, which increases proportionately. Indeed, as mentioned by Bischi et al. (2022), a high public indebtedness level and an increase in bond supply tend to raise interest rates, since investors raise the holding of bonds relative to money. Besides, for an indebted economy, an increase in bond supply also raises the risk of bankruptcy, pushing up the risk premium on the interest rate; the cost of the debt then increases exponentially the public debt.

In these conditions, a high public debt raises the interest rate and is harmful to economic activity. Indeed, it can favor sparing, and discourage on the contrary investment expenditure, interest-sensitive consumption expenditure. Government debt can 'crowd out' productive physical capital. The value of assets held by households is also decreased, which reduces private consumption through a wealth effect. Besides, a high public debt might cause the temptation, for a government, to monetize this public debt, causing inflation. In this context, the nominal interest rate must also rise, in order to minimize variations of the real interest rate. In this framework, nonlinearity between the sovereign risk premium and government indebtedness is widely identified in empirical literature.

For example, Alesina et al. (1992) measure this risk of default as the difference between the return from holding government debt and the return from holding 'safe' private debt of corresponding maturity. The authors find that this default risk is empirically very small, for 12 OECD countries over the period 1974-1989. The risk premium is positive only for countries with a high or rapidly growing public debt. Besides, they underline that the interest rate differential is positively correlated with the level of economic activity. In the same way, Bernoth et al. (2012) study bond yield differentials among EU government bonds issued between 1993 and 2005. They show that to fix risk premia, the start of EMU has shifted market attention to debt service payments, instead of liquidity conditions and width of the bonds' markets, in conformity with the situation in a monetary union as the United States. Markets pay less attention to public debt and fiscal deficit ratios, but they give more weight to the debt service ratio in their assessment of credit risk. Juessen et al. (2016) also find that interest rates on government bonds reflect expectations of the risk for a government to default. They show the existence of multiple equilibrium interest rates on government bonds that contain default risk premia. Sovereign default risk premia turn out to emerge at either very high debt to output ratios, or if the variance of productivity shocks is large.

Bi (2012) assesses that a nonlinear relation should be considered between sovereign risk premia and the level of government debt in equilibrium, consistent with the empirical evidence that once risk premia begin to rise, they do so rapidly. He develops a closed economy model to study the interactions among sovereign risk premia, fiscal policy and stochastic fiscal limits, which measure the ability and willingness of the government to service its debt. These fiscal limits arise endogenously from a dynamic Laffer curve: only a government which is on the 'slippery' side of the Laffer curve is unable to raise more tax revenue through higher tax rates. The distribution of fiscal limits is country-specific, depending on the size of the government, the degree of countercyclical policy responses, economic diversity, and political uncertainty, among other characteristics. The model rationalizes different sovereign ratings across developed countries. Alcidi and Gros (2019) also underline that if the risk premium depends on the debt level, the marginal cost of public debt is much higher than the interest rate on the public debt, which represents the average cost. Indeed, a country suffers a risk premium if the public debt is superior to a given level, for example 60% of GDP. Besides, they underline that the combination of a high debt level with a high risk

premium creates self-reinforcing loops. For example, in Italy, a high debt level combined with increasing deficits led to a higher risk premium, and hence to higher refinancing costs. On the contrary, in Portugal, a moderate reduction in fiscal deficits contributed to the fall of the risk premium to less than one half of the Italian level, thus reducing the interest payment burden, and improving the perspective of sustainability of the public debt.

More precisely, experience shows that, above a certain threshold of debt, the (market) interest rate on public debt increases with public indebtedness. This has been confirmed by many empirical studies. For example, Engen and Hubbard (2004) find that between 1976 and 2003, an increase in federal government debt equivalent to one percent of GDP would be expected to increase the long-term real interest rate by about three basis points. Laubach (2009) studies the relation between long-horizon expected government debt and deficits and expected future long-term interest rates, between 1976 and 2003. The estimated effects of government debt and deficits on interest rates are statistically and economically significant: a one percentage point increase in the projected deficit-to-GDP ratio is estimated to raise long-term interest rates (10-year Treasury bond rate) expected to prevail five years into the future by roughly 25 basis points. Similarly, a percentage point increase in the projected debt-to-GDP ratio raises future interest rates by only about 4 to 5 basis points.

In the same way, Ardagna et al. (2007) use a panel of 16 OECD countries between 1960 and 2002, in order to investigate the effects of government debts and deficits on long-term interest rates. They find that a one-percentage-point increase in the primary deficit relative to GDP increases nominal interest rates on 10-year government bonds by about 10 basis points. The effect of debt on interest rates is non-linear: only for countries with above-average levels of debt does an increase in debt affect the interest rate. While in a country with a debt-to GDP ratio of 119% a one-standard-deviation increase in government debt leads to an increase in the nominal interest rate on 10-year government bonds of about 36 basis points, an increase by the same amount where the public debt-to-GDP ratio is 58% leads to a 5 basis points decrease in the interest rate.

Bischi et al. (2022) also study the potential non-linearity linking the debt ratio to the real interest rate. They show that, in a dynamic framework, fiscal rules may not be enough to control the pattern of the debt ratio, and the adoption of a monetary policy, in the form of an interest rate rule, is necessary to control the pattern of the debt ratio for assuring its sustainability over time. Notably, the creation or disappearance of steady states, or periodic (stable) cycles, can generate scenarios of multi-stability. In their framework monetary and fiscal policies interact over time affecting both the debt ratio and GDP growth. The model allows for complex dynamics with stable and unstable equilibria that can even generate chaos and explosive patterns. According to the authors, mixed monetary and budgetary policies are thus necessary to stabilize the debt ratio. The higher the uncertainty about the debt sustainability in financial markets, the larger the chance the economy shifts towards instability. In line with this reasoning, our paper takes into account the non-linear relation between the public debt and the interest rate, and our goal is to put light on the interdependence between monetary and budgetary policies to define targeted levels of sustainability for the public debt.

3. Modelling the variation of the public debt

The goal of this paper is to study analytically the various factors influencing the variation and the sustainability of the public debt. We make the conventional hypothesis that the public debt consists of one-period bonds. The public debt in a given period (t) corresponds to the public debt inherited from the former period (t-1) increased by interest rates, and to the current budget deficit, the difference between public expenditure and tax resources. The variation of the public debt is then:

$$B_t = (1 + r_t)B_{t-1} + (G_t - T_t) \quad (1)$$

Therefore, in percentage of Gross Domestic Product (GDP), with lowercase letters:

$$\frac{B_t}{Y_t} = \frac{(1+r_t)B_{t-1}}{(1+y_t)Y_{t-1}} + \frac{Def_t}{Y_t} \quad (2)$$

$$b_t = \left(\frac{1+r_t}{1+y_t}\right) b_{t-1} + def_t \quad (3)$$

With, in period (t): (B_t): public debt at the end of the period; (b_t): public debt to GDP ratio; (r_t): nominal interest rate on the public debt; ($Def_t = G_t - T_t$): primary budget deficit (expenditures excluding interest payments minus revenues); (def_t): ratio of the primary deficit to GDP; (Y_t): nominal GDP; (y_t): nominal GDP growth rate; (G_t): primary (non-interest) budget expenditure; (T_t): total tax revenues.

3.1. A fiscal balance modelling the phenomenon of 'fiscal fatigue'

According to equation (3), the variation of the public debt to GDP ratio is as follows:

$$(b_t - b_{t-1}) = \left(\frac{r_t - y_t}{1+y_t}\right) b_{t-1} + def_t \quad (4)$$

The equilibrium and long-term public indebtedness level is then:

$$\bar{b} = \left(\frac{1+y}{y-r}\right) \overline{def} \quad (5)$$

- If (r>y), the maximal long-term public debt level which is sustainable would increase with and would be limited by the maximal budget surplus that a country can generate. Sustainability is then insured only if primary surpluses necessary to the financing of the debt never exceed a limit threshold considered as practicable1.
- If (r<y), the public debt automatically decreases and converges towards a positive and finite value increasing with the maximal conceivable budget deficit. Maintaining a constant positive debt ratio is then consistent with running primary deficits forever.

However, we can complicate the previous equations. Indeed, beyond the cyclical component of the budget balance depending on economic activity because of automatic stabilizers (β), the primary balance seems empirically to react to the public debt level. Bohn (1998) considers a simple linear equation, where the primary public balance increases after an increase of the public debt-to-GDP ratio, in order to ensure the sustainability of the public debt, as defined by the respect of the government intertemporal budget constraint. However, we can mostly consider a more complex cubic equation, where the budget deficit to GDP ratio varies as follows:

$$def_t = \overline{def} + \alpha_1 b_{t-1} - \alpha_2 b_{t-1}^2 + \alpha_3 b_{t-1}^3 - \beta y_t \quad (6)$$

Regarding the parameter (β), in developed countries, fiscal revenues usually move approximatively as GDP, as long as non-tax revenues are relatively low ($\Delta T_t \sim \Delta Y_t$). On the other side, public expenditure is quite independent from GDP, given the small share of unemployment relative expenditure [$\Delta\left(\frac{G_t}{Y_t}\right) \sim -\Delta Y_t = -y_t$]. So, the elasticity of the budget deficit to GDP ratio to a variation of economic activity is approximatively minus 0.5 [$\frac{\partial def_t}{\partial y_t} = \frac{1}{2}\frac{\partial (G_t/Y_t)}{\partial y_t} -$

[1] Escolano (2010) assumes that the 'modified golden rule' is verified in the long run, i.e.: $\frac{(r-y)}{(1+y)} \sim 0.01 > 0$.

$\frac{1}{2}\frac{\partial(T_t/Y_t)}{\partial y_t} = \frac{1}{2}\frac{\partial(G_t/Y_t)}{\partial y_t} = -0.5$]. So, efficient budget rules should sufficiently increase the budget surplus when economic activity is above its potential level (β=0.5), in order to create enough room of maneuver to allow a budget deficit in case of recession.

Furthermore, the recent literature has investigated empirically the possibility of non-linear fiscal behavior conditional on the level of debt. Indeed, Ghosh et al. (2013) underline the idea of 'fiscal fatigue', because the ability of a government to increase primary balances cannot keep pace with rising debt. So, the fiscal effort tends to be so large that it becomes untenable. As a result, the government faces an endogenous debt limit beyond which debt cannot be rolled over. Using data for 23 advanced economies over 1970–2007, they find evidence of a fiscal reaction function like the one mentioned in equation (6). Specifically, according to the authors, the relation is well approximated by a cubic (and not a linear) function: at low levels of debt, there is no (or even a slightly negative) relation between the primary balance and the debt. As public debt increases, the primary balance also increases, but the responsiveness eventually weakens (if the debt approaches 90-100% of GDP), and then actually decreases at very high levels of public debt (if the debt approaches 150% of GDP). Therefore, the parameters of equation (6) mentioned in Gosh et al. (2013) are the following:

$$(\alpha_1 = 0.2080)\ (\alpha_2 = 0.0032)\ (\alpha_3 = 0.00001)\ (\beta = 0.4974)$$

Ostry et al. (2010) also find that at very low levels of debt, there is little response of the primary balance to rising debt. As debt increases, the balance responds more vigorously, but eventually the adjustment effort weakens as it becomes increasingly more difficult to raise taxes or cut primary expenditures further. The primary balance could even decrease for extremely high public debt levels. The equation for the budget deficit could thus well be cubic as in equation (6), even if Larch et al. (2021), for example, only consider the public debt-to-GDP squared ratio (α_2) as determinant increasing the likelihood to run inappropriate pro-cyclical fiscal policies. Checherita-Westphal and Zdarek (2017) estimate a fiscal reaction function for 18 euro area countries for the period 1970-20132. In the Euro Area, they find that the primary balance improves by about 0.03–0.04 for every 1 percentage point increase in the debt-to-GDP ratio after controlling for other relevant factors. Besides, the positive reaction of primary surpluses to higher debt strengthened over the crisis: the 2008 crisis acted as a disciplining device. The coefficient of GDP growth (β=0.35) shows that the budgetary balance strongly improves with economic growth. Nevertheless, empirical data do not find significance of 'fiscal fatigue', of a non-linear relation between the public debt and the primary balance.

However, Combes et al. (2017) investigate the reaction of fiscal policy to the business cycle in a panel of 56 developed, emerging and developing economies over 1990–2011. While they strengthen the established finding that fiscal policy is counter-cyclical, they reveal a non-linear response of fiscal policy to the business cycle, conditional upon the outstanding debt stock. They find that when the public debt-to-GDP ratio goes beyond an endogenously estimated threshold of 87%, fiscal policy turns pro-cyclical. Similar results are reported by Medeiros (2012). He uses Vector Auto-Regression models and a panel of fiscal reaction functions to simulate debt ratios for fifteen EU Member States. His results then suggest that primary balances show 'fiscal fatigue' and partial mean reversion to historical trends: the primary balance tends to decline, although remaining positive, at (very) high levels of the public debt-to-GDP ratio. So, between 1976 and 2011, he finds the following coefficients:

$$(\alpha_1 = 0.1926)\ (\alpha_2 = 0.00306)\ (\alpha_3 = 0.0000127)\ (\beta = 0.499)$$

[2] An extensive enumeration of empirical studies about Fiscal Reaction Functions, and of the coefficient of the budgetary response to the public debt level, is provided in Checherita-Westphal and Zdarek (2017).

Fournier and Fall (2017) also find evidence of 'fiscal fatigue', for 31 OECD countries over the period 1985-2013, starting around a debt ratio of 120% of GDP with a twist around 170% of GDP (without Japan there is no evidence), while for the Euro area group (15 countries) two thresholds (at 152% and 167% of GDP) are identified. Nevertheless, these results seem to be driven by the inclusion of Greece, as when Greece is dropped, fiscal fatigue appears at a lower debt ratio around 120% of GDP.

In this framework, what are the conditions of sustainability of the public debt? By combining equations (4) and (6), we obtain:

$$(b_t - b_{t-1}) = \alpha_3\left(b_{t-1} - \overline{b_1}\right)\left(b_{t-1} - \overline{b_2}\right)b_{t-1} + \left[\frac{(r_t - r)}{(1+y_t)} - \frac{(1+r)(y_t - y)}{(1+y_t)(1+y)}\right]b_{t-1} + \overline{def} - \beta y_t \qquad (7)$$

$$\overline{b_1} = \frac{\alpha_2}{2\alpha_3} - \frac{1}{2}\sqrt{\frac{(\alpha_2^2 - 4\alpha_1\alpha_3)}{\alpha_3^2} - \frac{4(r-y)}{\alpha_3(1+y)}}$$

$$\overline{b_2} = \frac{\alpha_2}{2\alpha_3} + \frac{1}{2}\sqrt{\frac{(\alpha_2^2 - 4\alpha_1\alpha_3)}{\alpha_3^2} - \frac{4(r-y)}{\alpha_3(1+y)}}$$

We obtain the following variations of the public debt level:

- If $(0 < b_{t-1} < \overline{b_1})$: the public debt increases and converges towards $(\overline{b_1})$
- If $(\overline{b_1} < b_{t-1} < \overline{b_2})$: the public debt decreases and converges towards $(\overline{b_1})$
- If $(b_{t-1} > \overline{b_2} > 0)$: the public debt increases and can explode.

Therefore, $(\overline{b_1})$ appears as a stable equilibrium for the public debt, whereas $(\overline{b_2})$ is on the contrary an unstable equilibrium beyond which the public debt can explode.

3.2. Risk premium on the interest rate and public debt sustainability

To go beyond the previous analysis, we can precise the various factors influencing the risk premium on the public debt of a given country, beyond the risk-free interest rate on the bonds market. The main factor influencing this bond spread is default (or credit) risk. Default risk is related to the fiscal sustainability prospects of a given country, and to the probability of debtor non-compliance relative to its obligations. The relative risk aversion and the confidence of investors might also be influenced by credit rating announcements, especially for negative announcement downgrading the notation of the public debt of a country. Besides, the premium beyond the risk-free interest rate can also be a liquidity risk premium: a return for an asset that cannot be cashed in easily or quickly, associated with the possibility that the bonds market might face temporary shortages during a market stress situation. Indeed, for example, as the integration of European bonds markets is not complete, a phenomenon of 'flight-to-safety' may induce investors to invest in a 'safe haven', in a country where the bonds market is larger. However, today, liquidity risk premia have become less important than fiscal considerations to affect variations of interest rates on the public debts of European countries. We also make the hypothesis that investors' confidence is not affected by current GDP growth, as empirical studies show that it is only potential and expected output (quite uncertain) which can influence real bond spreads.

So, we suppose that default (credit) risk is the main factor influencing the risk premium: a high public debt level is a bad signal for financial markets, which can increase the interest rate on government bonds of a given country beyond the risk-free interest rate. The problem of a low interest rate environment for a long time (as from 2007 to 2021) is that it can undermine fiscal 'prudence', encourage profligacy, as a high public debt is then less damageable. Nevertheless, after the subprime crisis in 2007, interest rates became much more sensitive to the

indebtedness level of the European countries, and a non-linear and convex relation appears between the public debt and the risk premium on interest rates. The interest rate increases with the public debt, because to repay the public debt, the country could have to devalue the real value of its debt with unexpected inflation, and then, the nominal interest rate must be higher. Or investors are 'risk averse', and then, the risk premium is a measure of this risk aversion regarding the future solvability of the public debt in case of uncertainty.

Such a relation can also be supported by empirical data. For example, Baldacci and Kumar (2010) compute median data for all G20 countries and 11 other advanced and emerging economies over the period 1980–2008. Bond yields are based on residual of regressions of nominal 10-year bond yields on short-term rates and inflation with country-specific fixed effects. Then, they report the weak positive correlation between gross public debt and interest rates on this public debt (adjusted long-term bond yields). In particular, between 1990 and 2007, in these 31 countries, public debt levels decreased on average from above 60% of GDP until 45% of GDP, whereas interest rates decreased from 12.5% until 6%.

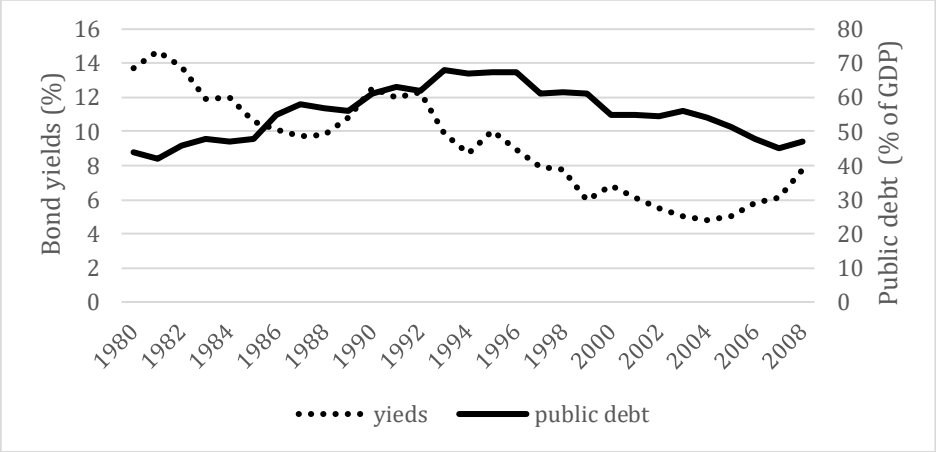

Figure 1. Public debt to GDP ratio (%) and adjusted long-term bond yields (%).

Source: Baldacci and Kumar (2010)

In this framework, Fournier and Fall (2017) find, for 31 OECD countries over the period 1985-2013, that current debt limits are high for most countries thanks to particularly low current interest rates. However, the authors show that the reaction of the fiscal balance to high public debt levels depends on the pressure of the real interest rate on the government, and they underline the vulnerability of governments to a change in macroeconomic conditions and to market reactions. The debt limit depends on confidence of investors, and therefore on the risk associated to the probability of default on a given public debt. They find that the interest rate increases with public debt; nevertheless, this increase is small and not always significant for non-Euro area OECD countries, while it is significant and larger for Euro area (and particularly for small) countries. So, we can make the hypothesis that the interest rate increases with the public debt, according to the following equation:

$$r_t = r_0 - \delta_1 b_{t-1} + \delta_2 b_{t-1}^2 \quad (8)$$

Where (r_0) is the risk-free nominal interest rate on the bonds market, and where the interest rate also includes a risk premium beyond this rate, according to the public debt level.

Fournier and Fall (2017) estimate that $(\delta_1 = -0.023)$ and $(\delta_2 = 0)$, a linear relation, for EMU member countries. Nevertheless, we can consider a more complex exponential relation on the model of equation (8). For example, De Grauwe and Ji (2013) confirm the hypothesis that government bonds markets in the Eurozone are more fragile and more susceptible to self-fulfilling liquidity crises than in stand-alone countries, especially since the debt

crisis in 2008. For the Eurozone, between 2000:Q1-2011:Q3, they find that ($\delta_1 = -0.0744$), or that ($\delta_1 = 0.0523$) and ($\delta_2 = 0.0009$) for a non-linear relation which is a better estimation. On the contrary, debt variables have a negligible influence in stand-alone countries. Furthermore, they authors find that a significant part of the surge in the spreads of the peripheral Euro-zone countries after 2010 would have been disconnected from underlying increases in the debt to GDP ratios and fiscal space variables and was only associated with negative self-fulfilling market sentiments.

Therefore, if we consider a non-linear increase of the interest rate according to the public debt, by combining equations (7) and (8), the variation of the public debt level is as follows:

$$(b_t - b_{t-1}) = \left(\alpha_3 + \frac{\delta_2}{(1+y)}\right)\left(b_{t-1} - \overline{b_3}\right)\left(b_{t-1} - \overline{b_4}\right)b_{t-1} - \frac{\delta_2(y_t - y)}{(1+y)(1+y_t)}b_{t-1}^3$$

$$+ \frac{\delta_1(y_t - y)}{(1+y)(1+y_t)}b_{t-1}^2 - \frac{(1+r_0)(y_t - y)}{(1+y_t)(1+y)}b_{t-1} + \overline{def} - \beta y_t \quad (9)$$

$$\overline{b_3} = \frac{(\alpha_2(1+y) + \delta_1)}{2(\alpha_3(1+y) + \delta_2)} - \frac{1}{2}\sqrt{\frac{(\alpha_2(1+y) + \delta_1)^2}{(\alpha_3(1+y) + \delta_2)^2} - 4\left(\frac{(\alpha_1(1+y) + r_0 - y)}{(\alpha_3(1+y) + \delta_2)}\right)}$$

$$\overline{b_4} = \frac{(\alpha_2(1+y) + \delta_1)}{2(\alpha_3(1+y) + \delta_2)} + \frac{1}{2}\sqrt{\frac{(\alpha_2(1+y) + \delta_1)^2}{(\alpha_3(1+y) + \delta_2)^2} - 4\left(\frac{(\alpha_1(1+y) + r_0 - y)}{(\alpha_3(1+y) + \delta_2)}\right)}$$

So, for plausible and moderate values of economic growth, we have:

- If ($0 < b_{t-1} < \overline{b_3}$): the public debt increases and converges towards ($\overline{b_3}$)
- If ($\overline{b_3} < b_{t-1} < \overline{b_4}$): the public debt decreases and converges towards ($\overline{b_3}$)
- If ($b_{t-1} > \overline{b_4} > 0$): the public debt increases and can explode.

Therefore, ($\overline{b_3}$) appears as a stable equilibrium for the public debt, whereas ($\overline{b_4}$) is on the contrary an unstable equilibrium beyond which the public debt can explode. Furthermore, as we will see in the following section 4, ($\overline{b_3}$) and ($\overline{b_4}$) are much lower than ($\overline{b_1}$) and ($\overline{b_2}$), and therefore, if the interest rate increases exponentially with the public debt, conditions of sustainability of the public debt could be much more difficult to comply with.

4. Conditions for public debt sustainability

To obtain significant results, we use the following calibration of the parameters of our model. Regarding 'fiscal fatigue' and the relation between the public debt and the primary budgetary deficit (both in % of GDP), we consider the parameters found in Ghosh et al. (2013): ($\alpha_1 = 0.2080$), ($\alpha_2 = 0.0032$) and ($\alpha_3 = 0.00001$). We consider the long-term risk-free nominal interest rate ($r_0 = 0.6\%$), and the long-term GDP growth ($y = 2\%$). Regarding the relation between the interest rate and the public debt, we consider the parameters found in De Grauwe and Ji (2013): ($\delta_1 = 0.0523$) and ($\delta_2 = 0.0009$). However, in this section, we will study the sensibility of our results to a variation of the value of these parameters.

4.1. A variable interest rate including an exponential risk premium on the public debt

If the interest rate is fixed and is risk-free, without problem of confidence of investors and exponential increase with the public debt level (δ1= δ2=0), the value ($\overline{b_2} = 238.57\%$ of GDP with our basic calibration) beyond which the public debt explodes and becomes unsustainable can be quite high (see Figure 2). So, if the interest rate doesn't vary with the public debt, a very high public indebtedness could be sustainable without risk of explosion. However,

if investors are reluctant to lend to a highly indebted country, and if the interest rate exponentially increases with the public debt (δ1>δ2>0), the maximal sustainable public debt level ($\overline{b_4}$ around 59% of GDP) is more constrained and much weaker.

In Figure 2, values for (δ1) and (δ2) are chosen in a bracket allowing an increasing and exponential variation of the interest rate with the public debt, according to equation (8). This exponential relation has then very strong implications for the optimal public debt levels. Indeed, if (δ1) and (δ2) increase, the equilibrium values ($\overline{b_3}$) and ($\overline{b_4}$) initially strongly decrease, before remaining much more stable. It means that it is this relation and this risk premium which is determinant and fundamental for the limit of the public indebtedness. For example, with ($\delta_1 = 0.012$) and ($\delta_2 = 0.0002$), the nominal interest rate on the public debt is limited to ($r_t = 6.20\%$) even for a public debt of ($b_{t-1} = 200\%$) of GDP. However, the maximal public debt level allowed to avoid an explosion of this public debt and which could be sustainable is already as low as ($\overline{b_4} = 55.69\%$) of GDP. The public debt is then already constrained to converge towards a level ($\overline{b_3}$) which is extremely limited and very weakly positive.

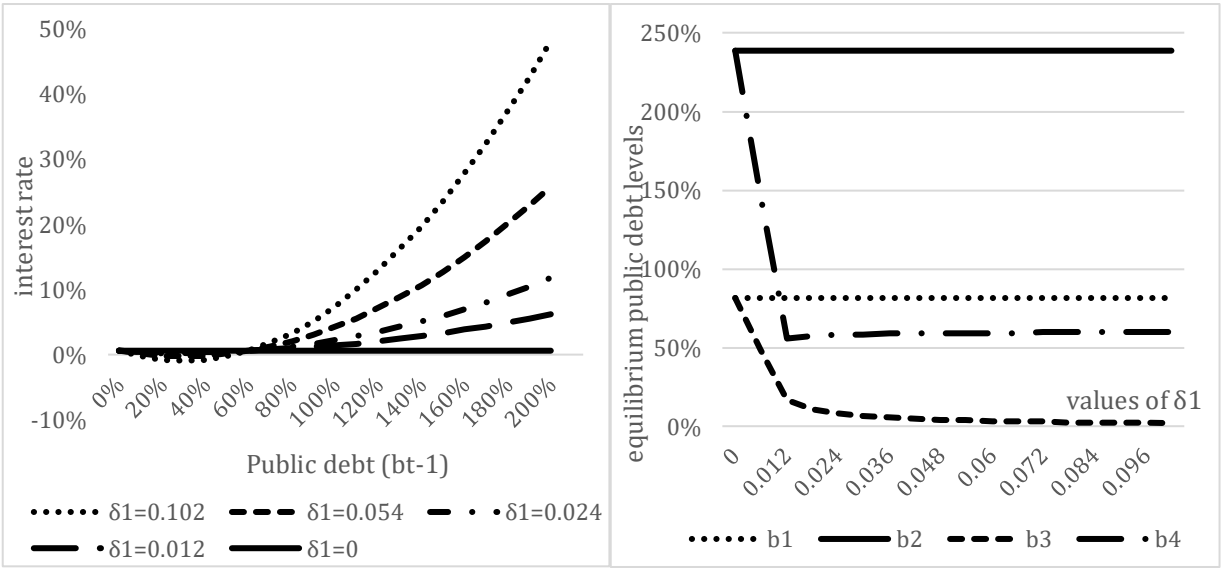

Figure 2. Effect of an interest rate varying with the public debt (risk premium) on the equilibrium levels of this public debt.

Calibration: ($r_0 = 0.6\%$) is the risk-free nominal interest rate. The risk premium is supposed to vary with the public debt level (default risk premium), and we suppose that both terms of the exponential function in equation (8) vary proportionately: ($\delta_2 = \delta_1/60$).

Therefore, the sustainable public debt level strongly depends on monetary policy. If the central bank can anchor expectations on a low interest rate, if credit rating agencies and private investors have confidence in the solvability of a country, a high public debt can be sustainable. On the contrary, episodes of financial stress and of lack of confidence in the fiscal situation of a country can imply movements of defiance, of increase in the default (credit) risk premium, of self-fulfilling expectations and of fast increase of interest rates making finally quickly the public debt unsustainable.

4.2. Long term economic activity and risk-free interest rate

Two factors are traditionally underlined in the economic literature, regarding the conditions of sustainability of the public debt. Indeed, these conditions are broadened if long-term economic activity (y) increases, or if the risk-

free nominal interest rate (r_0) decreases: then, ($\overline{b_1}$) is smaller and ($\overline{b_2}$) is higher. So, a high GDP growth rate and a low risk-free nominal interest rate are fundamental parameters to make the public debt sustainable, as mentioned in the economic literature. However, if the interest rate depends on the public debt level (δ1>δ2>0), the effects of (y) or of (r_0) are much more limited; ($\overline{b_4}$) only very slightly increases. So, the relation between the interest rate on the public debt and the risk premium in equation (8) strongly reduces the consequences of long-term economic activity or of the risk-free interest rate on the sustainable public debt.

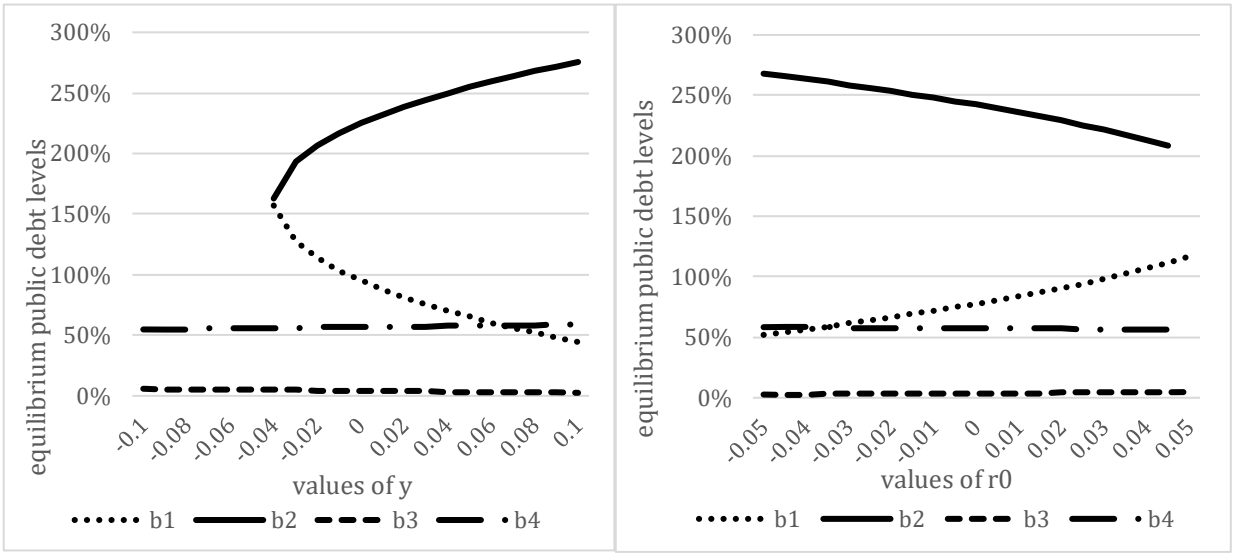

Figure 3. Effect of long-term economic growth and risk-free interest rate on the equilibrium levels of the public debt.

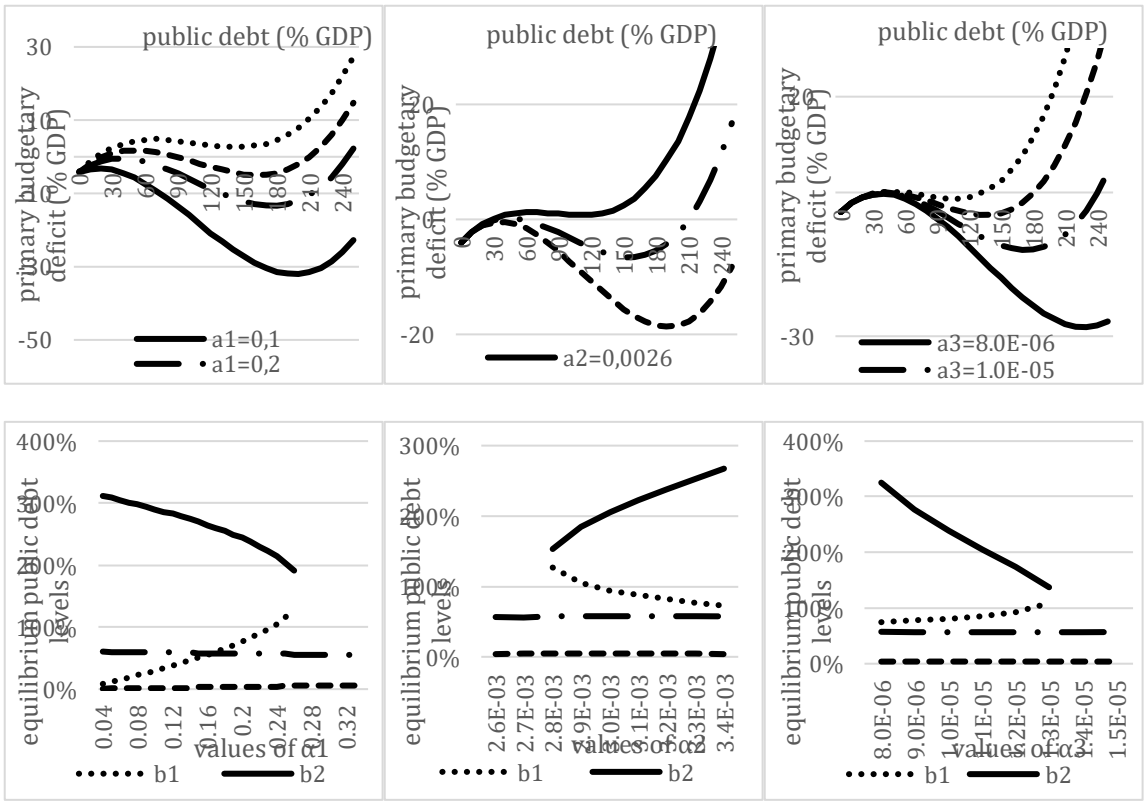

Figure 4. Effect of a primary budget balance varying with the public debt on the equilibrium levels of this public debt.

Therefore, the credibility of monetary policy to anchor expectations on a low interest rate, and the capacity to avoid the request by investors of a high default (credit) risk premium, strongly lessens the importance of traditional factors (risk-free interest rate or economic growth) to influence the sustainability of the public debt.

4.3. A budget balance depending on the public debt, with fiscal fatigue

In our model, a last parameter influences the sustainability of public indebtedness: the variation of the primary budget balance according to the public debt level. In Figure 4, values of (α1), (α2) and (α3) are chosen in a bracket allowing 'fiscal fatigue' for the budget deficit, as well as plausible values for this budget deficit.

A higher value of (α_1) or (α_3), or a smaller value of (α_2), could risk increasing the primary budget deficit which is allowed despite a high public indebtedness. Therefore, if the long-term nominal interest rate is fixed and risk-free, the fiscal space for the equilibrium public debt level should be reduced, and ($\overline{b_2}$) should decrease. On the contrary, for higher values of (α_1) or (α_3), or for a smaller value of (α_2), the stable long-term equilibrium public debt level ($\overline{b_1}$) would increase. However, if the interest rate depends on the public debt level (δ1>δ2>0), the consequences of the sensitivity of the primary budget balance to the level of the public debt become negligible; ($\overline{b_4}$) only very slightly decreases, while ($\overline{b_3}$) very slightly increases. So, the relation between the interest rate and the public debt in equation (8) strongly reduces the consequences of 'fiscal fatigue' on the sustainable public debt.

Therefore, the credibility of monetary policy to anchor expectations on a low interest rate, and the capacity to avoid the request by investors of a high default (credit) risk premium, could appear as more fundamental than fiscal factors, than the reaction of the primary fiscal balance to the public debt, to influence the sustainability of this public debt.

5. Conclusion

Traditionally, conditions of sustainability of the public debt have long been related quite exclusively to fiscal policy and to budgetary parameters: in particular, to the budget deficit and to economic growth. However, the interaction between fiscal and monetary policies regarding the fixation of the interest rate is fundamental. If the confidence of risk-adverse investors is important to avoid the explosion of the public debt, the sustainability of the public debt depends on the mix between budgetary and monetary policy, which interaction should obviously be taken into account.

Indeed, a simple analytical modelling shows that if the nominal interest rate increases exponentially with the public debt, because of a default (credit) risk premium, if the confidence of investors is a fundamental parameter for the weight of reimbursement of the public debt, the conditions of sustainability of the public debt could be much more difficult to comply with. This increasing and exponential relation, this risk premium, is the main determinant hardly constraining the maximal sustainable public debt level, beyond which public indebtedness could become unsustainable and would risk exploding. Indeed, if the interest rate is risk-free, values for which the public debt can be sustainable are less constraining if the long-term GDP growth rate is high, or if the risk-free long-term nominal interest rate is small. They are also less constraining if the sensibility of the budget deficit to the public debt level ('fiscal fatigue') is more limited: if the country even decides to turn to a non-negligible primary budget surplus in case of a high public debt. However, if the interest rate exponentially increases with the public debt level, in case of a significant importance of the default (credit) risk premium, these parameters have very limited consequences on sustainable and equilibrium public debt levels. The sustainable public debt that a government should target is then much smaller than in absence of this risk premium.

Our model only considers the default (credit) risk premium as determinant of the variation of the interest rate on the public debt beyond the risk-free interest rate. So, this model could be complexified and improved by taking into account other determinants of the risk premium, beyond the public debt level; or the fact that the confidence of investors may depend on future excepted economic growth in a given country. We leave these interrogations for potential future research.

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Author contributions

Conceptualization, Methodology, Formal Analysis: Séverine Menguy.

Declaration of Competing Interest

The author claims that the manuscript is completely original. The author also declares no conflict of interest.

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