Economic Analysis Letters 2023 2(2) 40-51

Economic Analysis Letters

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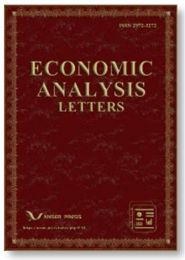


# Don't worry about the debt-GDP Ratio

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## ABSTRACT

I will show that if the propensity to consume from savings satisfies appropriate conditions, the debt-GDP ratio will not grow infinitely large and fiscal collapse will not occur. Using a basic macroeconomic model, with an overlapping generations model in mind, we show the following results: 1) The budget deficit including interest payments on the government bonds equals an increase in the savings from a period to the next period. 2) If the savings in the first period is positive, we need budget deficit to maintain full employment under constant prices or inflation in the later periods. 3) Under an appropriate assumption about the propensity to consume from savings, the debt-GDP ratio converges to a finite value. It does not diverge to infinity. The larger the propensity to consume from savings, the smaller the budget deficit required to achieve full employment. The larger the propensity to consume from savings, the less likely it is that the debt-GDP ratio will become large.

## KEYWORDS

Budget deficit; Debt-GDP ratio; Functional Finance Theory

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ISSN 2972-3272
doi: 10.58567/eal02020006

Received 30 March 2023; Accepted 2 May 2023; Available online 21 May 2023

## 1. Introduction

Japan's government debt exceeds 1,000 trillion yen, and the debt-GDP ratio is over 260%, and it is said that the debt-GDP ratio will grow without limit in the future, and Japan's public finances will eventually collapse. Oliver Blanchard (Blanchard, 2022) said: Debt becomes unsafe when there is a non-negligible risk that, under existing and likely future policies, the ratio of debt to GDP will steadily increase, leading to default at some point. The natural way to proceed is then straightforward. The dynamics of the debt ratio depend on the evolution of three variables: primary budget balances (that is, spending net of interest payments minus revenues); the real interest rate (the nominal rate minus the rate of inflation); and the real rate of economic growth.

As Blanchard notes, many discussions of debt-GDP ratio use simple calculations based on comparisons of primary budget balances, the real interest rate, and the real economic growth rate. But is the argument not so simple? Assuming a steady state of full employment, which may or may not include inflation, the size of the budget deficit to achieve this is naturally determined, and the larger the budget deficit is, the higher the inflation rate is. On the other hand, the larger (smaller) the ratio of the portion of savings or assets held by consumers that is spent on consumption, i.e., the propensity to consume from savings and the larger the propensity to consume from income, the smaller (larger) the budget deficit required to achieve full employment under a constant rate of price increase. Therefore, the larger the propensity to consume from savings is, the less likely it is that the debt-GDP ratio will become large, and the less likely it is that its value will diverge indefinitely. Based on these considerations, this paper shows that if the propensity to consume from savings satisfies appropriate conditions, the debt-GDP ratio will not grow infinitely large and fiscal collapse will not occur.

One of the most commonly used conditions for examining fiscal stability is the so-called Domar condition (Domar, 1944). The Domar condition compares the interest rate with the economic growth rate under balanced budget (excluding interest payments on the government bonds), and if the former is greater than the latter, public finance will become unstable, and the debt-GDP ratio will continue to grow. Yoshino and Miyamoto, 2020) try to modify the Domar condition by focusing not only on the supply side of government bonds but also on the demand side, while keeping the idea of fiscal instability indicated by the Domar condition. However, my interest is different. I consider a problem of the debt-GDP ratio from the perspective of Functional Finance Theory (Lerner, 1943, 1944) and MMT (Modern Monetary Theory, Kelton, 2020), Mitchell, Wray, and Watts (2019, Wray, 2015[1]) using a simple macroeconomic model. I will show that under an appropriate assumption about the propensity to consume from savings the Domar condition is meaningless.

The paper also takes into account that the budget deficit is financed not only by the issuance of government bonds but also by the issuance of money, i.e., seigniorage. In the latter case, the possibility of fiscal collapse is even smaller.

In the next section, I examine the relation between the budget deficit and the debt-GDP ratio and will show the following results.

(1) The budget deficit including interest payments on the government bonds equals an increase in the savings from a period to the next period.

(2) If the savings in the first period is positive, the budget deficit is necessary to maintain full employment under constant prices or inflation in the later periods.

(3) Under an appropriate assumption about the propensity to consume from savings, the debt-GDP ratio converges to a finite value. It does not diverge to infinity even if the fiscal expenditure is financed solely by government bond. If the fiscal expenditure is financed solely by money not government bond, the debt-GDP ratio cannot diverge even when the propensity to consume from savings is very low.

[1] Japanese references of MMT are Mochizuki (2020), Morinaga (2020), Nakano (2020), Park (2020), Shimakura (2019).

In Section 3, I examine the Domar condition and will show that under an appropriate assumption about the propensity to consume from savings it is meaningless.

## 2. Budget deficit and debt-GDP ratio

Using a simple macroeconomic model, I analyze budget deficit and the debt-GDP ratio. Savings are made by government bonds and money. The amounts of government bonds and money supply are determined by the government. Although money does not earn interest and government bonds earn interest, consumers are willing to hold a certain amount of money for reasons such as the liquidity of money. The holding of money is a decreasing function of the interest rate of the government bonds. This part implicitly assumes an overlapping generations model in which people live for two periods[2]. People decide how much money and government bonds to hold so that the marginal utility of holding one more unit of money and the marginal utility of interest income from holding government bonds are equalized. Since the marginal utility of money decreases as the amount of money held increases, the amount of money held is a decreasing function of the interest rate of the government bonds.

The share of government bonds in savings is denoted by $b(r), 0 < b(r) \leqq 1$. $r$ is the interest rate of the government bonds. The share of money in savings is $1 - b(r)$. The interest rate of the government bonds is determined by the monetary policy of the government.

### *2.1. Period 0*

First consider Period 0 at which the world starts. Variables other than $P_0$ are nominal values. $P_0$ is the price level in Period 0. Let $Y_0$, $C_0$, $I_0$, $T_0$ and $G_0$ be the GDP, consumption, investment, tax and fiscal expenditure in Period 0. Then,

$$Y_0 = C_0 + I_0 + G_0$$

The consumption is

$$C_0 = \bar{C}_0 + \alpha(Y_0 - T_0)$$

$\bar{C}_0$ is the constant part of consumption in Period 0. $\alpha$ is the propensity to consume from income. $0 < \alpha < 1$. $\bar{C}_0$ is financed by the savings carried over from the previous period. Since there is no previous period of Period 0,

$$\bar{C}_0 = 0$$

Then,

$$C_0 = \alpha(Y_0 - T_0)$$

and

$$Y_0 = \alpha(Y_0 - T_0) + I_0 + G_0$$

From this

$$(1 - \alpha)(Y_0 - T_0) = I_0 + G_0 - T_0$$

The savings in Period 0, which is carried over to the next period, is

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[2] In some other studies, which are according to the model by M. Otaki such as Otaki (2007, 2009, 2015), we use an overlapping generations model to analyze the problem of budget deficit in a growing economy.

$$S_0 = (1-\alpha)(Y_0 - T_0) - I_0$$

Therefore, we have

$$G_0 - T_0 = (1-\alpha)(Y_0 - T_0) - I_0 = S_0$$

Let us assume full employment in Period 0, and denote the full employment real GDP by $Y_f$, that is,

$$Y_0 = P_0 Y_f$$

Then, we obtain

$$S_0 = (1-\alpha)(P_0 Y_f - T_0) - I_0$$

and

$$G_0 - T_0 = (1-\alpha)\left(P_0 Y_f - T_0\right) - I_0 = S_0 \quad (1)$$

This is the budget deficit needed to achieve full employment in Period 0. It is determined by $Y_f$, $I_0$, $T_0$ and $P_0$. From this we get the following equation.

$$G_0 = (1-\alpha)(P_0 Y_f - T_0) + T_0 - I_0$$

This is the fiscal expenditure needed to achieve full employment given $T_0$, $I_0$ and $P_0$. If the budget deficit is larger than the value in (1) given $T_0$ and $I_0$, then $P_0$ increases by inflation and (1) still holds. The debt to GDP ratio in Period 0 is

$$\frac{S_0}{Y_0} = \frac{(1-\alpha)(P_0 Y_f - T_0) - I_0}{P_0 Y_f}$$

This is decreasing in $\alpha$, which is the propensity to consume from income.

### *2.2. Period 1*

Next, I consider Period 1. All variables other than $P_0$ represent nominal values. Let $Y_1$, $C_1$, $I_1$, $T_1$ and $G_1$ be the GDP, consumption, investment, tax and fiscal expenditure in Period 1. Then,

$$Y_1 = C_1 + I_1 + G_1$$

The consumption is

$$C_1 = \bar{C}_1 + \alpha(Y_1 - T_1)$$

$\bar{C}_1$ is the constant part of consumption in Period 1. It is financed by the savings carried over from Period 0. Let $r_0$ be the interest rate of the government bonds, which is carried over from Period 0 to Period 1. Then, we can write

$$\bar{C}_1 = \delta(1 + b(r_0)r_0)S_0, 0 < \delta < 1$$

and

$$C_1 = \delta(1 + b(r_0)r_0)S_0 + \alpha(Y_1 - T_1)$$

$\delta$ is the propensity to consume from savings. I assume $0 < \delta < 1$. Thus,

$$Y_1 = \delta(1 + b(r_0)r_0)S_0 + \alpha(Y_1 - T_1) + I_1 + G_1$$

From this

$$(1-\alpha)(Y_1-T_1)=\delta(1+b(r_0)r_0)S_0+I_1+G_1-T_1$$

Therefore,

$$G_1-T_1=(1-\alpha)(Y_1-T_1)-I_1-\delta(1+b(r_0)r_0)S_0$$

The savings in Period 1, which is carried over to Period 2, is

$$S_1=(1-\alpha)(Y_1-T_1)-I_1+(1-\delta)(1+b(r_0)r_0)S_0$$

This means

$$G_1-T_1=S_1-(1+b(r_0)r_0)S_0$$

Alternatively,

$$G_1-T_1+b(r_0)r_0S_0=S_1-S_0 \tag{2}$$

I assume that the economy grows by population growth (or technological progress). The real growth rate is $g>0$. Also, the prices may rise from Period 0 to Period 1, that is, there may be inflation. Let $p$ be the inflation rate. Then,

$$(1+g)(1+p)-1=g+p+gp$$

is the nominal growth rate.

Under nominal growth at the rate of $g+p+gp$,

$$Y_1=(1+g)(1+p)P_0Y_f$$

Tax and investment also increase at the same rate as follows when inflation is predicted,

$$T_1=(1+g)(1+p)T_0, \qquad I_1=(1+g)(1+p)I_0$$

Then, the savings in Period 1 is

$$S_1=(1-\alpha)(1+g)(1+p)(P_0Y_f-T_0)-(1+g)(1+p)I_0+(1-\delta)(1+b(r_0)r_0)S_0$$

It is rewritten as

$$S_1=(1+g)(1+p)S_0+(1-\delta)(1+b(r_0)r_0)S_0 \tag{3}$$

Since $\delta<1$, we have

$$S_1>(1+g)(1+p)S_0 \tag{4}$$

From (2) and (3),

$$G_1-T_1+b(r_0)r_0S_0=(1+g)(1+p)S_0+[b(r_0)r_0-\delta(1+b(r_0)r_0)]S_0$$

and

$$G_1-T_1=(1+g)(1+p)S_0-\delta(1+b(r_0)r_0)S_0<(1+g)(1+p)(G_0-T_0)$$

They are budget deficits, with or without interest payments on the government bonds needed to achieve full employment in Period 1 under nominal growth at the rate of $g+p+gp$. The larger the value of $\delta$, the smaller the the value of $G_1-T_1$. Thus, the larger the propensity to consume from savings, the smaller (larger) the budget deficit required to achieve full employment. Note that $S_0$ is decreasing in $\alpha$. Therefore, $G_1-T_1$ is decreasing in both $\alpha$

and δ.

Without inflation (nor deflation) we have

$$G_1 - T_1 + b(r_0)r_0S_0 = (1+g)S_0 + [b(r_0)r_0 - \delta(1+b(r_0)r_0)]S_0$$

and

$$G_1 - T_1 = (1+g)S_0 - \delta(1+b(r_0)r_0)S_0$$

## 2.3. Period 2

Next, consider Period 2. Also in this subsection all variables other than $P_0$ represent nominal values. Let $Y_2$, $C_2$, $I_2$, $T_2$ and $G_2$ be the GDP, consumption, investment, tax and fiscal expenditure in Period 2. Then,

$$Y_2 = C_2 + I_2 + G_2$$

The consumption is

$$C_2 = \bar{C}_2 + \alpha(Y_2 - T_2)$$

$\bar{C}_2$ is the constant part of consumption in Period 2. Let $r_1$ be the interest rate of the government bonds, which is carried over from Period 1 to Period 2. Similarly to the case of Period 1,

$$\bar{C}_2 = \delta(1+b(r_1)r_1)S_1$$

and

$$C_2 = \delta(1+b(r_1)r_1)S_1 + \alpha(Y_2 - T_2)$$

Then,

$$Y_2 = \delta(1+b(r_1)r_1)S_1 + \alpha(Y_2 - T_2) + I_2 + G_2$$

From this

$$(1-\alpha)(Y_2 - T_2) = \delta(1+b(r_1)r_1)S_1 + I_2 + G_2 - T_2$$

Therefore,

$$G_2 - T_2 = (1-\alpha)(Y_2 - T_2) - I_2 - \delta(1+b(r_1)r_1)S_1$$

The savings in Period 2, which is carried over to Period 3, is

$$S_2 = (1-\alpha)(Y_2 - T_2) - I_2 + (1-\delta)(1+b(r_1)r_1)S_1$$

This means

$$G_2 - T_2 = S_2 - (1+b(r_1)r_1)S_1$$

Alternatively,

$$G_2 - T_2 + b(r_1)r_1S_1 = S_2 - S_1 \tag{5}$$

Again, we suppose that the economy nominally grows at the rate of $g + p + gp$, then

$$Y_2 = (1+g)^2(1+p)^2P_0Y_f$$

We assume that the inflation rate $p$ is constant. Tax and investment also increase at the same rate as follows,

$$T_2 = (1+g)^2(1+p)^2 T_0, I_1 = (1+g)^2(1+p)^2 I_0$$

Then, the savings in Period 2 is

$$S_2 = (1-\alpha)(1+g)^2(1+p)^2(P_0Y_f - T_0) - (1+g)^2(1+p)^2 I_0 + (1-\delta)(1+b(r_1)r_1)S_1$$

It is rewritten as

$$S_2 = (1+g)^2(1+p)^2 S_0 + (1-\delta)(1+b(r_1)r_1)S_1 \tag{6}$$

Since $\delta < 1$ and

$$S_1 > (1+g)(1+p)S_0$$

assuming

$$r_0 \approx r_1 \tag{7}$$

we have

$$S_2 > (1+g)(1+p)S_1 \tag{8}$$

From (5) ,

$$G_2 - T_2 + b(r_1)r_1S_1 = (1+g)^2(1+p)^2 S_0 + [b(r_1)r_1 - \delta(1+b(r_1)r_1)]S_1 \tag{9}$$

and

$$G_2 - T_2 = (1+g)^2(1+p)^2 S_0 - \delta(1+b(r_1)r_1)S_1 \tag{10}$$

Since

$$G_1 - T_1 = (1+g)(1+p)S_0 - \delta(1+b(r_0)r_0)S_0$$

by the assumption of (7), we obtain

$$G_2 - T_2 < (1+g)(1+p)(G_1 - T_1)$$

(9) and (10) are budget deficits, with or without interest payments on the government bonds, needed to achieve full employment in Period 2.

By (3) and (6),

$$\begin{aligned} S_2 = [&(1+g)^2(1+p)^2 + (1-\delta)(1+b(r_1)r_1)(1+g)(1+p) \\ &+(1-\delta)^2(1+b(r_0)r_0)(1+b(r_1)r_1)]S_0 \end{aligned} \tag{11}$$

Without inflation (nor deflation),

$$G_2 - T_2 + b(r_1)r_1S_1 = (1+g)^2 S_0 + [b(r_1)r_1 - \delta(1+b(r_1)r_1)]S_1$$

and

$$G_2 - T_2 = (1+g)^2 S_0 - \delta(1+b(r_1)r_1)S_1$$

## *2.4. Period 3 and beyond*

From now on, for simplicity, the interest rates in all periods are equal. Also in this subsection all variables represent nominal values, and the inflation rate is constant. It may be zero. Denote the interest rate by $r$. By similar reasoning, for Period 3 we get

$$G_3 - T_3 = S_3 - (1 + b(r)r)S_2$$

and

$$G_3 - T_3 + b(r)rS_2 = S_3 - S_2 \tag{12}$$

The savings in Period 3 is

$$S_3 = (1 + g)^3(1 + p)^3S_0 + (1 - \delta)(1 + b(r)r)S_2 \tag{13}$$

Thus,

$$G_3 - T_3 + b(r)rS_2 = (1 + g)^3(1 + p)^3S_0 + [b(r)r - \delta(1 + b(r)r)]S_2 \tag{14}$$

and

$$G_3 - T_3 = (1 + g)^3(1 + p)^3S_0 - \delta(1 + b(r)r)S_2 \tag{15}$$

(14) and (15) are budget deficits, with or without interest payments on the government bonds, needed to achieve full employment in Period 3.

From (11) and (13), we get

$$S_3 = [(1 + g)^3(1 + p)^3 + (1 - \delta)(1 + b(r)r)(1 + g)^2(1 + p)^2$$
$$+(1 - \delta)^2(1 + b(r)r)^2(1 + g)(1 + p) + (1 - \delta)^3(1 + b(r)r)^3]S_0$$

Proceeding with this argument, we obtain the following result for Period $n$, $n \geqq 1$.

$$G_n - T_n + b(r)rS_{n-1} = S_n - S_{n-1} \tag{16}$$

With or without inflation, we have

$$S_n = [(1 + g)^n(1 + p)^n + (1 - \delta)(1 + b(r)r)(1 + g)^{n-1}(1 + p)^{n-1}$$
$$+ \cdots + (1 - \delta)^{n-1}(1 + b(r)r)^{n-1}(1 + g)(1 + p) + (1 - \delta)^n(1 + b(r)r)^n]S_0 \tag{17}$$

Similarly, for Period $n - 1$,

$$S_{n-1} = [(1 + g)^{n-1}(1 + p)^{n-1} + (1 - \delta)(1 + b(r)r)(1 + g)^{n-2}(1 + p)^{n-2}$$
$$+ \cdots + (1 - \delta)^{n-2}(1 + b(r)r)^{n-2}(1 + g)(1 + p) + (1 - \delta)^{n-1}(1 + b(r)r)^{n-1}]S_0 \tag{18}$$

(2), (5), (12) and (16) mean that the budget deficit including interest payments on the government bonds equals an increase in the savings from a period to the next period. From (17) and (18),

$$S_n > (1 + g)(1 + p)S_{n-1} \tag{19}$$

Thus, by (4), (8) and (19), with (2), (5), (12) and (16),

**Proposition 1**: We need budget deficit (including interest payments) to maintain full employment under constant prices or inflation in each period.

## *2.5. Debt-GDP ratio*

Since

$$Y_n=(1+g)(1+p)Y_{n-1}$$

(17) and (18) mean

$$\frac{S_n}{Y_n}-\frac{S_{n-1}}{Y_{n-1}}=\left(\frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)}\right)^n\frac{S_0}{Y_0}=\left(\frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)}\right)^n\frac{(1-\alpha)(P_0Y_f-T_0)-I_0}{P_0Y_f}$$

Since $(1-\delta)(1+b(r)r)>0$,

$$0<\frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)}<1 \tag{20}$$

is equivalent to

$$b(r)r-\delta(1+b(r)r)<g+p+gp$$

or

$$\delta>\frac{b(r)r-g-p-gp}{1+b(r)r} \tag{21}$$

$g+p+gp$ is the nominal growth rate. $0\leqq b(r)\leqq 1$ and $r$ is the interest rate of the government bonds. I assume $r\leqq 1$, which means that the interest rate is not larger than 100%.

Since $\delta$ is the propensity to consume from savings, we can assume $\delta>\frac{1}{2}$. Then, (20) and (21) are reduced to

$$\frac{1}{2}(b(r)r-1)<g+p+gp$$

This is definitely satisfied with positive growth rate. If it is satisfied, when

$$n\to+\infty,\qquad \frac{S_n}{Y_n}-\frac{S_{n-1}}{Y_{n-1}}\to 0$$

From (17), we obtain

$$\frac{S_n}{Y_n}=\left[1+\frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)}+\cdots+\left(\frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)}\right)^{n-1}+\left(\frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)}\right)^n\right]\frac{S_0}{Y_0}$$

If

$$0<\frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)}<1$$

we get

$$\frac{S_n}{Y_n}\to\frac{1}{1-\frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)}}\frac{S_0}{Y_0}=\frac{1}{1-\frac{(1-\delta)(1+b(r)r)}{(1+g)(1+p)}}\frac{(1-\alpha)(P_0Y_f-T_0)-I_0}{P_0Y_f}$$

The debt-GDP ratio $\frac{S_n}{Y_n}$ converges to a finite value. Note that it is decreasing in $\alpha$ and $\delta$. It does not diverge to infinity. Summarizing the result,

**Proposition** 2: Under an appropriate assumption about the propensity to consume from savings, the debt-GDP ratio converges to a finite value. It does not diverge to infinity. The limit value of the debt-GDP ratio is decreasing in both the propensity to consume from income, $\alpha$, and that from savings, $\delta$.

Consider some other cases. Assume

$$b(r) = 0.9, \qquad g + p + gp = 0.2, \qquad r = 0.4$$

Then, for (20) and (21) to be satisfied, we need

$$\delta \geq 0.118$$

When

$$b(r) = 0.8, \qquad g + p + gp = 0.2, \qquad r = 0.5$$

we need

$$\delta \geq 0.143$$

These are very weak conditions. In this case, even if $b(r) = 1$, that is, the fiscal expenditure is fianced solely by government bond, we need only the following condition.

$$\delta \geq 0.2$$

If $b(r) = 0$, that is, the fiscal expenditure is fiananced solely by money, (21) is reduced to

$$\delta > -\frac{g + p + gp}{1 + b(r)r}$$

This will surely be fulfilled.

## 3. About Domar condition

As we describe in the next section, the interest rate can be determined by monetary policy so that the so-called Domar condition (Domar(1944), that the interest rate must be less than the economic growth rate to prevent the debt-GDP ratio from becoming infinitely large (if a balanced budget is achieved excluding interest payments on government bonds), can be satisfied, but even if this condition is not satisfied, the debt-GDP ratio will not become infinitely large. When savings are made in both government bonds and money, the issue is not the interest rate on government bonds itself, but the product of the share of savings held in government bonds and the interest rate on government bonds $b(r)r$. We call

$$\delta(1 + b(r)r) - 1 \tag{22}$$

the adjusted interest rate. Since $\delta < 1$ and $b(r) \leqq 1$, It is not larger than $r$.

Let us assume balanced budget excluding interest payments on the government bonds in Period 1 as follows.

$$G_1 - T_1 = 0$$

Then, (2) and (3) mean that the following equation must hold.

$$(1 + g)(1 + p) = \delta(1 + b(r_0)r_0) \tag{23}$$

If

$$1 + g < \delta(1 + b(r_0)r_0)$$

there is excess demand for goods. Then, the prices rise and the nominal growth rate $g+p+gp$ equals

$$\delta(1+b(r_0)r_0)-1$$

For periods after Period 1 we obtain similar results.

By (23), (20) is rewritten as

$$0<\frac{1-\delta}{\delta}<1$$

If $\delta>\frac{1}{2}$, it is satisfied. Then,

$$\frac{S_n}{Y_n}-\frac{S_{n-1}}{Y_{n-1}}<\frac{S_0}{Y_0}$$

and when

$$n\to\infty,\qquad \frac{S_n}{Y_n}-\frac{S_{n-1}}{Y_{n-1}}\to 0$$

Therefore, the debt-GDP ratio can not diverge to infinity even if

$$1+g<\delta(1+b(r_0)r_0)$$

or

$$g<\delta(1+b(r_0)r_0)-1$$

Summarizing the result,

**Proposition** 3: Even if the adjusted interest rate (22) is larger than the real growth rate under balanced budget excluding interest payments, if the appropriate assumption about the proportion of the savings consumed holds, the debt-GDP ratio can not diverge to infinity.

## 4. Determination of interest rate

The demand for money in Period 0 is

$$(1-b(r_0)S_0$$

Denote the money supply by $M_0$. Then, $r_0$ is determined so that

$$(1-b(r_0)S_0=M_0$$

is satisfied. Similarly, let $M_n$ be the money supply in Period $n$. Then, the interest rate in Period $n$ is determined so that

$$(1-b(r_n)S_n=M_n$$

is satisfied. As the money supply $M_n$ increases, $r_n$ and $b(r_n)$ must be smaller. Therefore, an increase in the money supply lowers the interest rate, and also interest payment $b(r_n)r_nS_n$ decreases. This is the effect of monetary policy.

## 5. Conclusion

I have argued that fiscal collapse is impossible because the debt-GDP ratio can not diverge to infinity under an appropriate condition about the propensity to consume from savings. Mainly I have shown that if the propensity to consume from savings is not so small, the debt-GDP ratio converges to a finite value, and it does not diverge to infinity. As I stated in the introduction of this paper, the larger (smaller) the propensity to consume from savings (and income), the smaller (larger) the budget deficit required to achieve full employment under a constant rate of price increase. Therefore, the larger the propensity to consume from savings (and income) is, the less likely it is that the debt-GDP ratio will become large, and the less likely it is that its value will diverge indefinitely. It is not simply a matter of considering the relationship among the interest rate, the growth rate, and the primary budget deficit, as Blanchard (2022) suggests.

## Funding Statement

This research received no external funding.

## Conflict of interest

All the authors claim that the manuscript is completely original. The authors also declare no conflict of interest.

## Acknowledgment

I am very grateful to the anonymous referee for his helpful discussions and suggestions.

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