Economic Analysis Letters 2023 2(2) 17-22

Economic Analysis Letters

Homepage: https://anser.press/index.php/EAL

An Examination of Ramsey's Critique: The Lack of Support in Dismissing Keynes's Logical Theory of Probability

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ABSTRACT

In his 1922 review for Cambridge Magazine of Keynes's A Treatise on Probability, and again in his 1926 review "Truth and Probability" (published in 1931 and republished in Kyburg and Smokler's 1980 edition), Ramsey provided examples critiquing Keynes's logical theory of probability. However, these examples do not disprove Keynes's theory as all of them are flawed. The flaws arise because all the pairs of propositions that Ramsey used to substantiate his criticisms are irrelevant to each other. Such irrelevant propositions are explicitly excluded from Keynes's theory in pages 4-6 of his A Treatise on Probability. This paper highlights that Ramsey's examples, which involve irrelevant propositions, do not disprove Keynes's Boolean, relational logic. The only other defense of Keynes's theory, put forth by Watt in 1989, was found to be deficient by Brady in 2022.

KEYWORDS

Boolean logic; relational; propositional logic; premises; conclusions; argument form; conditional probability

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ISSN 2972-3272
doi: 10.58567/eal02020003

Received 25 March 2023; Accepted 7 May 2023; Available online 15 May 2023

1. Introduction

This paper substantiates the validity of Keynes's relational, propositional logic as applied in chapters I, II, IV, V, VI, X-XVII, XX, XXII, XXVI, XXIX, XXX, and XXXIII of his A Treatise on Probability (TP, 1921). It conclusively argues against the dismissal of Keynes's theory based on Ramsey's critiques, suggesting they lack substantial backing.

Additionally, the paper delves into other aspects of Keynes's theory. This includes its implications for the field of probability theory, its influence on academic teaching and study of the subject, and its potential applications in real-world scenarios. In these discussions, the paper addresses Keynes's interval approach to probability and critiques of the mathematical expectations concept, which presupposes precise probability.

2. Method-Examining Ramsey's basic claims in 1922

Ramsey's initial statement in his 1922 paper—"Mr. Keynes takes probabilities or probability relations as indefinable and says that if q has to p the probability relation of degree a, then knowledge of p justifies rational belief of degree a in q." (Ramsey, 1922, p.3)—provides an incomplete representation of Keynes's position. This is due to Ramsey's omission of Keynes's requirement that propositions p and q must be related or associated to form an argument. According to Keynes, the argument form necessitates that one proposition (premises, denoted by h) provide relevant evidence for the second proposition (conclusion, denoted by a). Importantly, the argument can involve more than one premise or conclusion. It's not restricted to a single h proposition and a single a proposition as Ramsey posits, without citation to A Treatise on Probability. Keynes outlines his argument form as (a/h).

In the third paragraph of Ramsey's note, he claims, "First, he (author's note-Keynes) thinks that between any two non-self-contradictory propositions there holds a probability relation (Axiom I)..." (Ramsey, 1922, pp.3-4). However, no writings by Keynes during his lifetime support this assertion (see Brady, 2004a, ; Brady, 2022a). Ramsey repeats the same error he made in his opening paragraph, overlooking the specific argument form that the propositions must adopt to comply with Keynes's definition on pages 4-6 of TP.

Ramsey's claim doesn't align with Keynes's application of propositions, which need to be articulated as arguments (Keynes, 1921, p.4). One proposition must contain relevant evidence, while the second proposition must be a conclusion related to the evidence-bearing proposition. Only under these conditions is a relation of logical probability present. Keynes never stated in his A Treatise on Probability or in any other work during his lifetime that "…between any two non-self-contradictory propositions there holds a probability relation (Axiom I) …" (Ramsey, 1922, p.3).

There is no such Axiom I in Keynes's A Treatise on Probability. Consequently, Ramsey's example—"My carpet is blue" and "Napoleon was a great general"—is an oxymoron. His two propositions do not form an argument (see Keynes, 1921, p.4), making his example incorrect.

Moreover, Keynes would never assert as Ramsey suggests: "...it is easily seen that it leads to contradictions to assign the probability 1/2 to such cases, and Mr. Keynes would conclude that the probability is not numerical." (Ramsey, 1922, p.3). In contrast to Ramsey's assertion, Keynes would conclude that the probability is not defined.

Keynes elaborates on the application of his Principle of Indifference on pages 52-56 of his book. Ramsey's two propositions are unrelated to each other. Keynes explicitly states that if one proposition, let's say x, is irrelevant to another, let's say y, then y is also irrelevant to x. Therefore, it's impossible to assign a probability of 1/2 to each proposition or any other probability (Keynes, 1921, p.55). Russell's demonstration that Keynes's logic refutes Ramsey (Russell, 1922, p.120, footnote *) and Brady's additional arguments (Brady, 2022b, c) further support this.

3. Discussion

3.1 Ramsey's 1926 version of his 1922 review

Ramsey's three examples in 1926 include: "Besides this view is really rather paradoxical; for any believer in induction must admit that between 'This is red' as conclusion and 'This is round', together with a billion propositions of the form 'a is round and red' as evidence…" (Ramsey, 1980, p.28; Kyburg and Smokler, 1980). This example directly contravenes the argument form outlined by Keynes on pages 4-6 of TP. The propositions "This is red" and "This is round" share no relationship, meaning neither provides any information or evidence about the other. There is no conditional probability for (this is red/given that that is round). Given the example's irrelevance to Keynes's book, it can only be deemed erroneous. Also, note Ramsey's futile attempt to introduce a joint probability ("and") unrelated to Keynes's theory of conditional probability.

Ramsey also provides another example: "If, on the other hand, we take the simplest possible pairs of propositions such as 'This is red' and 'That is blue' or 'This is red' and 'That is red', whose logical relations should surely be easiest to see, no one, I think, pretends to be sure what is the probability relation which connects them." (Ramsey, 1926; In Kyburg and Smokler, 1980, p.28). Just like his 'My carpet is blue' and 'Napoleon was a great general' example, and the 'this is red' and 'this is round' example, Ramsey's 'This is red' and 'That is blue' example fails due to the lack of relationship between the two propositions. Conversely, his 'This is red' and 'That is red' proposition pair is related. All probabilities are conditional for Keynes, so we can recast this as the conditional probability that 'this is red', given that 'that is red'. 'This is red' is the conclusion (a proposition), and 'that is red' is the available evidence (h proposition) with a probability $\geq 1/2$, as the only other possible conclusion, not red, lacks supporting evidence. In Brady (2004a, b), it was demonstrated that Ramsey's "or" can be replaced with an "and", leading to the conclusion that the conditional probability (this is red, given that is red) exceeds the conditional probability (this is red, given that is blue).

Consider Ramsey's final example concerning a coin toss: "It is true that about some particular cases there is agreement, but these, paradoxically, are always immensely complicated; we all agree that the probability of a coin coming down heads is 1/2, but we can none of us say exactly what is the evidence which forms the other term for the probability relation about which we are then judging." (Ramsey, 1980, p.28; Kyburg and Smokler, 1980).

The issue here is that Ramsey neglects to question whether the coin is fair or not. By framing the problem as calculating a marginal probability, $P(H)=1/2$, he obscures the evidence, as opposed to considering P (H/the coin is fair) =1/2, which is a conditional probability.

Therefore, we should revise Ramsey's claim, "...we all agree that the probability of a coin coming down heads is 1/2," to, "...we all agree that the probability of a FAIR coin coming down heads is 1/2," where the word "fair" implicitly contains the evidence.

Ramsey's assertion that we all agree that a coin coming up heads has a probability of 1/2 is incorrect because the probability of an UNFAIR coin coming up heads IS NOT 1/2. It is only accurate to conclude that the probability of a FAIR coin coming up heads is 1/2.

3.2 Implications and Applications of Keynes's Probability Theory in Academic and Real-World Contexts

The prevailing belief is that Ramsey's critiques refuted Keynes's logical theory of probability, which was subsequently replaced by the subjectivist theory of probability. The following quotations illustrate this perception. The first comes from Franklin (2001), a defender of the logical theory of probability, while the next two are from Gillies (1972) and Sahlin (1991), both supporters of the subjectivist theory. The last quotation is from Gerrard (2023), who contends that Keynes, in response to Ramsey's critique, capitulated and repudiated his own theory: "...it is necessary to briefly consider F. P. Ramsey's criticisms of Keynes, which have been influential in convincing many that no logical theory of probability is possible...Ramsey's fundamental criticism is that 'there really do not seem to be any such things as the probability relations he [Keynes] describes. He supposes that, at any rate in certain cases, they can be perceived; but speaking for myself I feel confident that this is not true. I do not perceive them, and if I am to be persuaded, they exist it must be by argument; moreover, I shrewdly suspect that others do not perceive them either, because they are able to come to so very little agreement as to which of them relates any two given propositions.' (Ramsey 1926, p. 57)." (Franklin, 2001, p.289; italics added).

The subjective theory of probability surfaced as an answer to specific issues within the traditional logical view of probability. In the logical account, evidence 'e' justifies a single rational degree of belief in a hypothesis or prediction 'h.' This is the logical probability of 'h' given 'e.' Probability is a relationship we can 'cognize correctly,' as Keynes phrases it ([1920], p. 5).

However, the problem, as Ramsey argues ([1926], p. 161), is that 'there do not seem to be any such things as the probability relations Keynes describes' (Gillies, 1972, p.138). Ramsey's fundamental objection against Keynes is that this probability relation appears non-existent. He insists that he cannot perceive it and suspects no one else can (Sahlin, N-E, 1991).

Moreover, Gerrard (2023, p.199) notes that Ramsey is technically correct that Keynes's first axiom permits a potential probability relation between two unrelated propositions. Gerrard also agrees with Ramsey's claim that 'there do not seem to be any such things as the probability relations he described' (1926, [1990]. p.57).

Subsequently, Gerrard asserts that 'Keynes repudiates the analytic project of the Treatise to construct a logical theory of probability as a set of objective relations between propositions' (Gerrard, 2023, p.209).

However, this argument overlooks George Boole's accomplishment in his 1854 work 'The Laws of Thought,' where he formulated a logical theory of probability using his relational, propositional logic (Boole, 1854, chapters I, XI, XII, XVI-XXI).

The primary flaw in Ramsey's critique, highlighted in italics in the Franklin quotation above, is that Keynes's theory deals with specific sets of related propositions, not Ramsey's 'any two given propositions.'

This situation presents a significant philosophy of science issue: Ramsey never disproved Keynes's theory of probability, an imprecise, interval-valued theory using lower and upper bounds. Assuming academics can address the challenge of rehabilitating a theory never genuinely refuted, Keynes's approach offers practical applications. As Franklin points out, it can apply to initial and a priori probabilities with limited evidentiary support (low weight): 'If one represents probabilities of low weight by imprecise numbers, calculations may take more trouble. But there are no major technical difficulties. The simplest method is to use probabilities with error bounds (Walley 1991, Kyburg, 1974, chs. 9–10), but one can also calculate with fuzzy probabilities if desired (Pan and Yuan 1997; cf. Levi 1974, section IV).' (Franklin, 2001, p.284). Spiegelhalter, et al., 1993, echo the same point.

Keynes's interval-valued probability theory could provide valuable insights into the ongoing controversy regarding the correct or incorrect use of mathematical expectation calculations. Specifically, it could help explain how businesspeople form expectations of future returns to justify investment decisions in long-term, durable capital goods projects. This controversy originates from Keynes's discussion on "reasonable versus unreasonable calculations" in the General Theory (GT; 1936, pp.161-163). Reasonable calculations, according to Keynes, would be based on imprecise, inexact methods involving approximations (GT, pp.39-40, 43-44), as opposed to precise mathematical expectations, which he deemed unreasonable. Therefore, Keynes supported the use of reasonable, albeit inexact or imprecise calculations, over precise mathematical expectations.

4. Results

This paper's primary conclusion is that Ramsey failed to uncover any significant logical, methodological, philosophical, or epistemological issues with Keynes's logical theory of probability. This is contrary to Ramsey's own assertions and the claims made by D. Gillies. Gillies suggested, "The subjective theory of probability emerged as a solution to certain problems that had developed in the older logical view of probability. In this logical account, a body of evidence 'e' justifies a single rational degree of belief in a hypothesis or prediction 'h'. This is the logical probability of 'h' given 'e'. Probability is a relation we can 'cognize correctly', to use Keynes's term ([1920], p. 5). However, as Ramsey states ([1926], p. 161), 'there really do not seem to be any such things as the probability relations he (Keynes) describes.' Attempts were indeed made to evaluate the supposed logical probabilities using the 'principle of indifference', but the principle proved insurmountably challenging. Another shortcoming was the justification of the probability axioms based solely on unsatisfactory appeals to 'logical intuition'. Given these issues, the founders of the subjective theory (Ramsey and de Finetti) independently proposed that the seemingly non-existent objective degrees of rational belief should be replaced" (Gillies, 1972, p.138).

However, Gillies seems to overlook the fact that Keynes's logical theory of probability does not rest on the principle of indifference or the additivity assumption of the pure mathematical laws/axioms of the probability calculus. Instead, it's grounded in Keynes's adaptation of Boole's interval-valued theory of probability, found in his 1854 work, The Laws of Thought (Boole, LT, pp.265-268). Keynes first introduced this theory in Chapter III of A Treatise on Probability, further developing it mathematically in Part II, Chapters XV and XVII.

5. Conclusion

Ramsey's critique of Keynes's logical theory of probability is fundamentally flawed. He misleadingly asserts that Keynes's theory should be applicable to "any two" propositions, including both relevant and irrelevant ones. However, all of Ramsey's examples involve pairs of irrelevant propositions. In contrast, Keynes's theory only applies to sets of propositions that are relevant to each other. Given Keynes's use of a Boolean relational, propositional logic, it should have been evident to Ramsey that the propositions need to be related (Keynes,1921, pp.4-6; Boole,1854, pp.1-10). Yet Ramsey's 1922 example of 'My carpet is blue, Napoleon was a great general' is repeatedly cited as discrediting Keynes's theory. See Misak (2020, p.114) and Misak (2022, p.9) for examples.

A recent paper by Gerrard (2023, pp.199-200,201,209) echoes all of Ramsey's original errors from 1922 and 1926. Gerrard's entire paper rests on Ramsey's misguided use of irrelevant propositions when discussing Keynes's theory, with his main example being 'My carpet is blue, Napoleon was a great general' (Gerrard,2023, p.199).

I predict that philosophers of science will face significant challenges in the future to satisfactorily explain how Ramsey's deeply flawed reviews led to the dismissal of Keynes's logical theory of probability. It is only after addressing this issue that Keynes's theory could be taught in university-level probability courses and subsequently applied.

Funding Statement

This research received no external funding.

Acknowledgments

I thank the referee for his point that the paper needed to broaden the discussion of Keynes’s theory, its application and its future potential.

Conflict of Interest

The author claims that the manuscript is completely original. The author also declares that there is no conflict of interest.

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